From the Research Laboratories

ON THE STRUCTURE AND EVOLUTION OF TWO-DIMENSIONAL SOAP FOAM, ENCLOSED BETWEEN GLASS SHEETS

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Abstract. Liquid foams are objects of great practical significance - they appear in firefighting mixtures, soaps, alcoholic drinks, etc. However, our knowledge about them is extremely limited due to their complexity. This investigation, based on the first problem of the International Young Naturalists Tournament (IYNT) 2019, aims to fill some of the gaps in our understanding of foams in general and of soap foams, enclosed between heavy sheets in particular. This article describes the main parameters of such a foam, the basic qualitative theory that governs its structure and an extensive experimental investigation concerning the evolution with time of this structure. Relevant parameters (e.g. concentration of the soap solution, distance between the glass plates) have been varied in order to find how the distribution of bubbles by size and other quantities behave.
Keywords: soap foam; structure; evolution; experimental investigation; IYNT

1. Introduction

In this section we will discuss the basic theory and definitions regarding this study.

1.1. Definition for a two-dimensional foam.

A foam will be called two-dimensional[1)] if it comprises only one layer of bubbles, as in Figure 1. A foam that consists of more than one layer is three-dimensional (Figure 2).

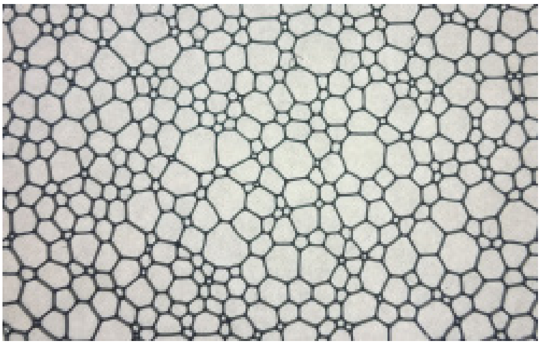


Figure 1. A 2D foam

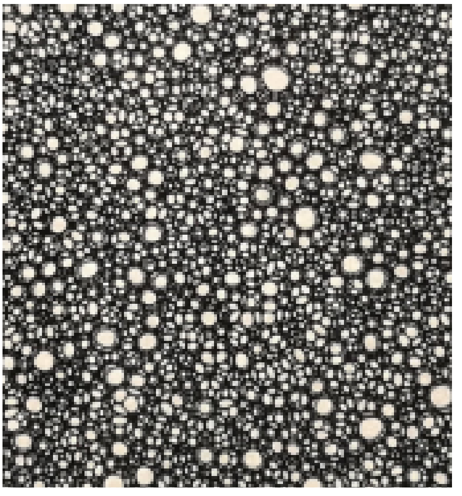


Figure 2. A 3D foam

1.2. Describing the structure of the foam.

This section aims to introduce the main quantities used to describe foam structure throughout the study.

1.2.1. Area distribution of the bubbles. Each bubble is characterised by its *area*[2)] S. To describe the overall structure of a foam we will use the distribution of the bubbles by their area. In other words, our aim is to express the number of bubbles ΔN within a certain area interval ΔS around S as a function of S. It is reasonable to assume that this distribution is the same for all parts of the foam, as the environmental conditions (e.g. temperature) are almost the same in all parts of the foam. This lets us build the distribution by analysing only a small area of the foam, which is technically easier and reduces experimental error. In order to be able to compare distributions for different foams we need to normalise ΔN, i.e. to investigate $\frac{\Delta N}{N}$ as a function of S, where N is the total number of bubbles in the analysed region of the foam. To summarise, we expect to obtain the following:

$$\frac{\Delta N}{N} = f(S)\Delta S$$

where $f(S)$ is determined experimentally.

1.2.2. Average area of the bubbles. Another quantity that we will use is the arithmetic mean (average) area of the bubbles in a given sample:

$$S_{avg} = \frac{1}{N}\sum_{i=1}^{N} S_i$$

where N is again the total number of bubbles in the sample and S_i is the area of the *i*-th bubble.

1.2.3. Standard deviation of the bubble areas. In this case the standard deviation of the bubble areas for a given sample is:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(S_i - S_{avg})^2}$$

1.3 Describing the evolution.

To describe the evolution with of the foam we will investigate how the quantities mentioned above vary with time.

1.4 Geometry of the foam.

In this section we will briefly discuss certain geometrical aspects of the structure of a 2D foam.

1.4.1 Plateau borders. Each two bubbles in a foam are separated by a part of a soap film, called *Plateau border* (Figure 3) (Mancini, 2005).

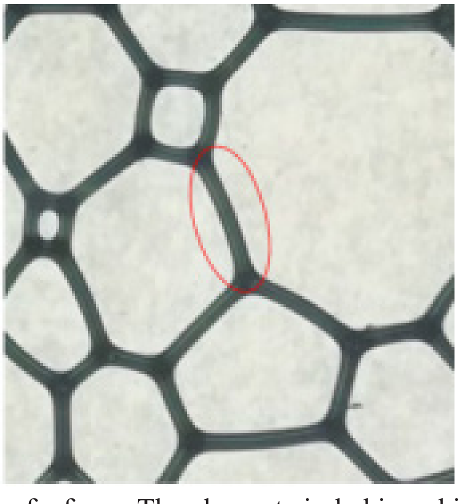


Figure 3. Segment of a foam. The element circled in red is a Plateau border.

1.4.2. Plateau's laws. Plateau's laws were discovered experimentally in the 19. century by the Belgian physicist Joseph Plateau (Mancini, 2005). They describe the shape of a *stable* foam and are a very good approximation for an almost stable[3)] one. The following two of them are relevant to this study:

1. Soap films comprise smooth and unbroken surfaces.
2. Soap films always intersect in threes at 120° angles.

2. Experimental investigation

In this section we will describe the setup and data acquisition techniques.

2.1. Soap solution

To produce the foam a mixture of liquid soap and water was used. The water used was from the tap rather than distilled, however the concentration of impurities is negligibly small. The masses of the soap and the whole solution are measured, allowing us to calculate the mass concentration of the soap:

$$w = \frac{m_{soap}}{m}$$

2.2. Foam production

The original idea was to produce the foam directly between the glass sheets using a syringe. However, it turned out to be unsuccesful. That is why the following procedure was used:

1. The soap solution is poured in a water bottle and shaked until a suffi cient amount of foam forms.
2. The excess solution that has not formed a foam is removed.
3. The foam is coloured using cooking dyes, in order to improve contrast (Figure 4).

Figure 4. A foam, coloured with blue cooking dye, that is ready to use in experiments

2.3. The setup

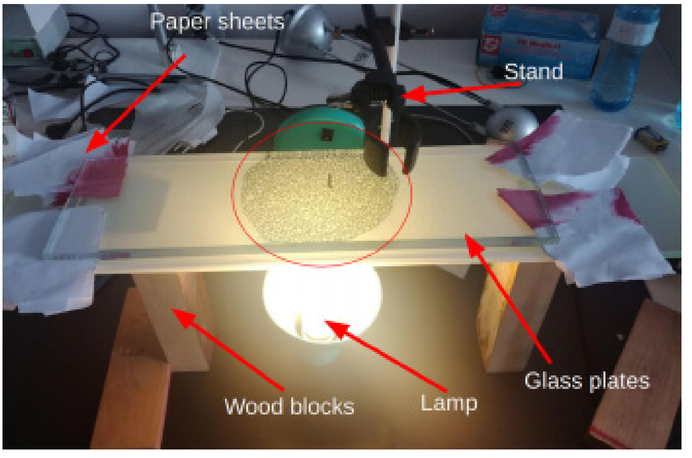


Figure 5. The Experimental setup. Circled in red in the center of the picture is the foam itself

The foam is put between two heavy glass sheets, the distance between which is controlled using sheets of paper. This construction is then placed on a frosted glass, covered with baking paper (Figure 6) and on top of wooden blocks. The foam is lit from underneath and the light gets diffused by the frosted glass and the baking paper. A digital camera (smartphone) is put on a stand, parallel to the glass sheets.

In some cases we would like to investigate the behaviour of a foam when the glass sheets are inclined relative to the ground (but still parallel to each other) (Figure 7). Then the wood blocks are replaced with other objects and the resultant inclination is measured with a level. The camera is also inclined in such a way that it remains parallel to the setup.

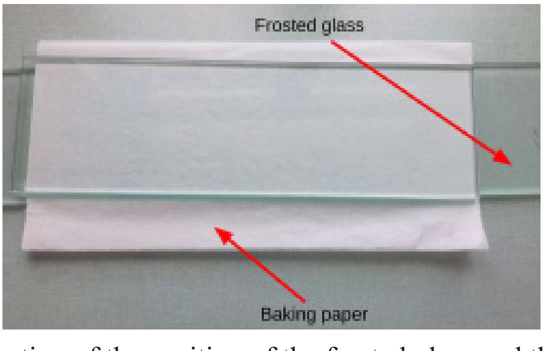


Figure 6. Illustration of the position of the frosted glass and the baking paper

Figure 7. An inclined setup

2.4. Experimental procedure

Here we will describe how the experiments were conducted.

2.4.1. For a small[4] inclination of the plates. The foam is photographed at some time interval (about 5-10 minutes, not necessarily constant throughout the experiment). After about 2 hours[5] the foam is removed.

2.4.2. For a high inclination of the plates. At high inclination the evolution is much faster and the foam is photographed using a time-lapse video.

2.5. Data analysis

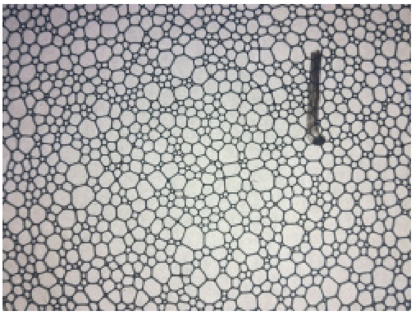


Figure 8. A typical photo, obtained with the described procedure. Notice the calibration line near the top right corner of the frame

The areas of individual bubbles (excluding those on the edges of the frame) are measured using the image editing software *ImageJ*. For videos *Tracker* is used to measure the velocities of individual bubbles. The calculations are then performed using *LibreOffice Calc*. For plots and curve fitting *QtiPlot* is used.

It is important to note that in every experiment near the center of the glass sheet there is a calibration line of known length (usually 1 or 2 cm), drawn with marker. It is used to set the scale in ImageJ and Tracker and calculate the actual sizes of bubbles.

3. Results

Here we will discuss the findings of this study in detail.

3.1. General observations

1. The main cause for the evolution of a foam is its *drying*, the water in the soap films dissipates with time (Figure 9).
2. The shape of the bubbles depends on the amount of the liquid in the foam. When it is big, the bubbles are round (figure 10a) and when it is small they are polygonal (Figure 10b).
3. Plateau's first law is obviously always valid.
4. Plateau's second law is valid only in the case of a ”dry” foam (small amount of liquid) (Figure 11).

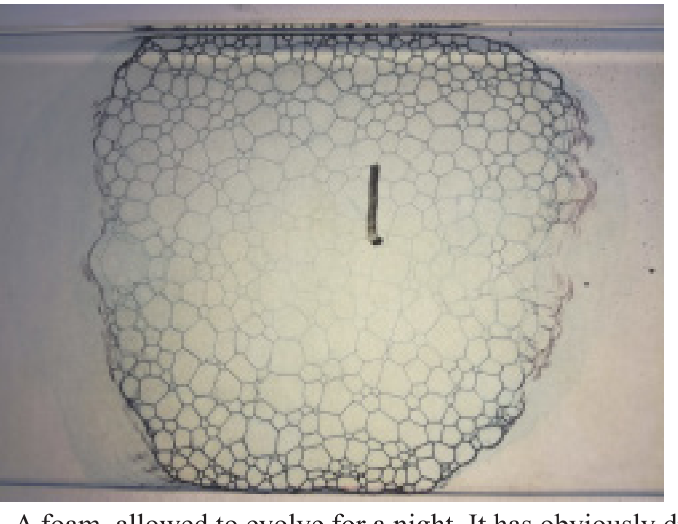
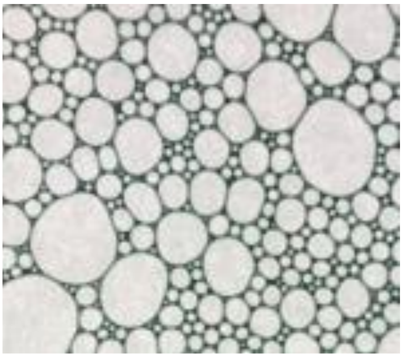


Figure 9. A foam, allowed to evolve for a night. It has obviously dried out

(a) “Wet” foam.



(b) “Dry” foam.

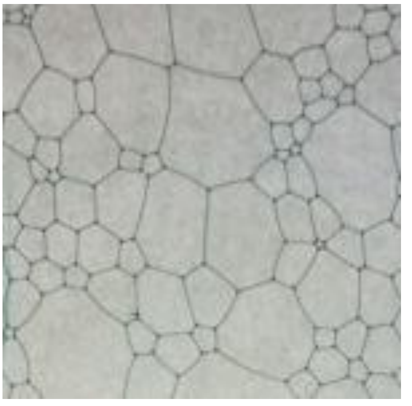


Figure 10. Comparison between foams with a different amount of liquid

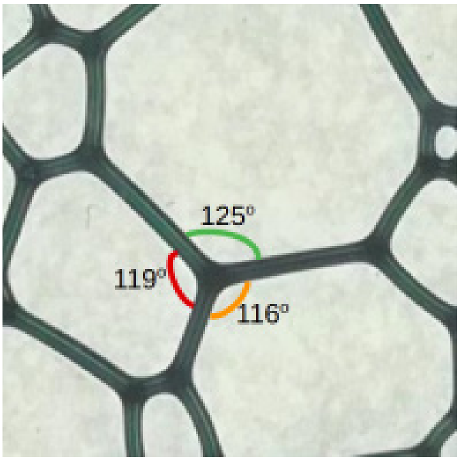


Figure 11. The angles of intersection at a random intersection point in a dry foam tend to 120°

3.2. Area distribution

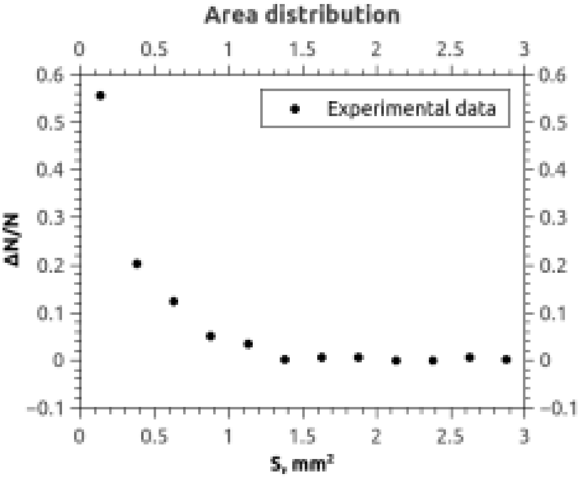


Figure 12. An experimentally obtained area distribution for mass concentration w = 19% and distance between the glass sheets d = 0.55 mm

Using the described procedure it was discovered that regardless of the parameter values the shape of the area distribution is *the same* in all experiments. The best-fit curve for it turned out to be a *decaying exponent* (Figure 13).

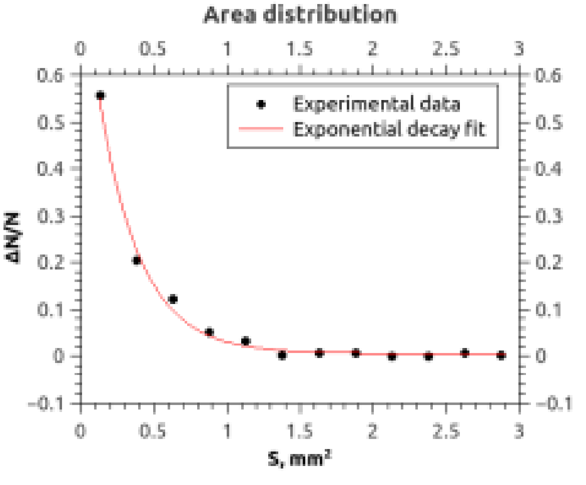


Figure 13. The same distribution with its best-fit curve.

The mathematical representation of this discovery is the following:

$$\frac{\Delta N}{N} = Ae^{-\frac{S}{S_0}}\Delta S,\ \ f(S) = Ae^{-\frac{S}{S_0}}$$

where A and S_0 are positive constants. S_0 has the following mathematical interpretation - if we increase the considered area with S_0, $\frac{\Delta N}{N}$ will decrease e times.

3.3. Evolution

Now we will look in more detail at the evolution of the foam.

3.3.1. Evolution of the area distribution

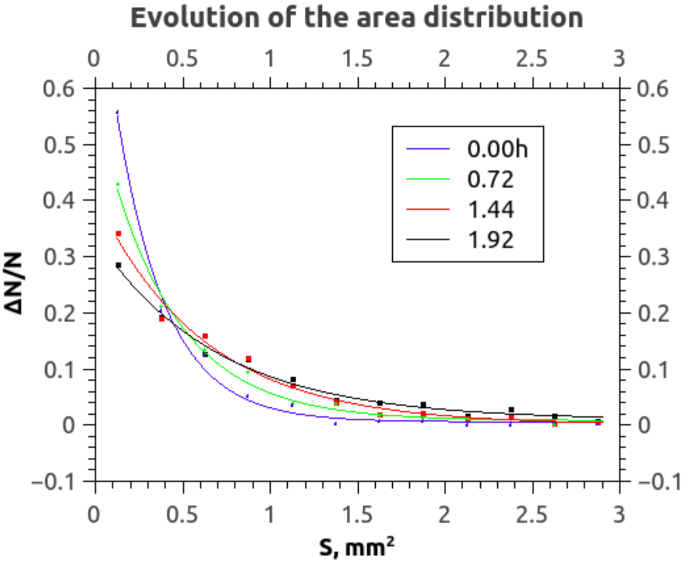


Figure 14. The area distribution at different moments of time for $w = 19\%$ and $d = 0.551$ mm

As you can see in Figure 14, the distribution becomes less steep at later time moments, i.e. A decreases and S_0 increases. The physical interpretation of this is that with time the bubble sizes tend to become more and more different.

3.3.2. Evolution of the area standard deviation. It turns out that standard deviation of the bubble areas grows approximately linearly with time (Figure 15). This supports the conclusion that we made in the previous section.

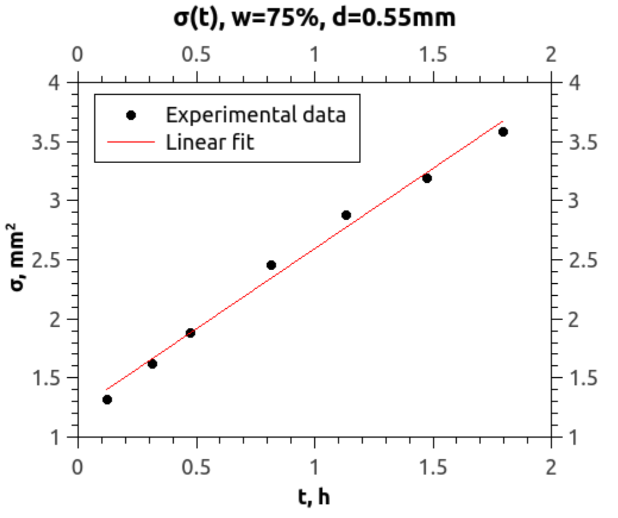


Figure 15. Graph of the standard deviation as a function of time

3.3.3. Evolution of the average area. The average area of the bubbles also appears to increase linearly with time (Figure 16). This means that the bubbles tend to become bigger with time.

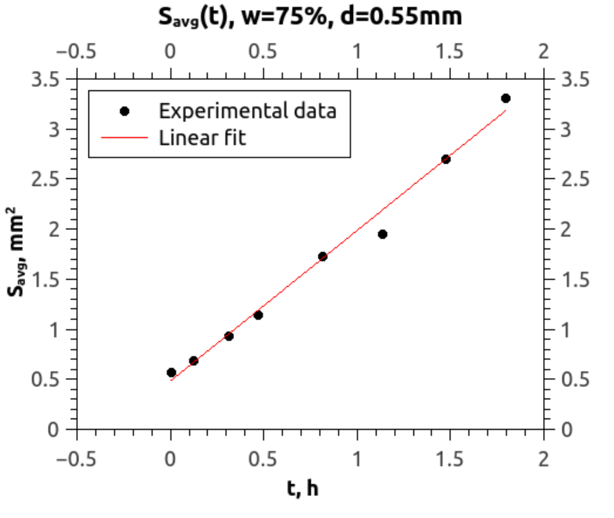


Figure 16. Graph of the average area as a function of time

3.4. Effect of the concentration

The mass concentration of the soap in the soap solution was varied in a large range of values (19 – 75%). The authors of this paper did not manage to find any clear correlation between it and the properties of the foam.

3.5. Effect of the distance between the sheets

Experiments have been conducted for three different distances between the glass sheets - 0.11 mm, 0.33 mm and 0.55 mm. The only clear correlation that could be found based on this is that the initial average size of the bubbles decreases with it as shown in Figure 17.

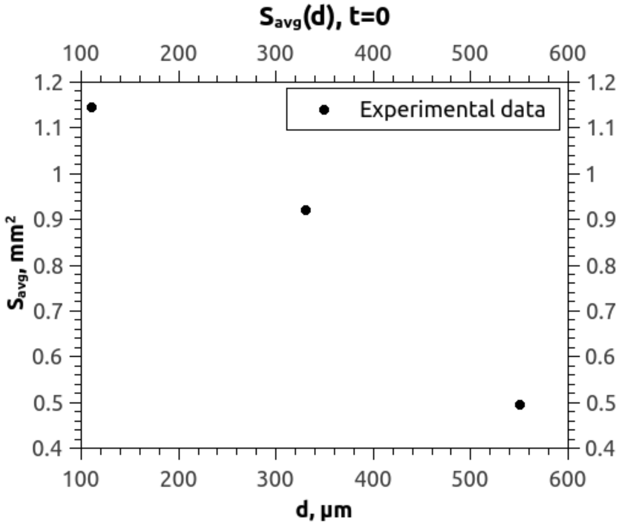


Figure 17. Relation between the initial average area and the distance between the plates for w = 54%

3.6. Observations on the behaviour at small inclinations

In the range 3° – $20^\circ$the evolution unfolds similarly for all inclinations. In the first minutes a large part of the liquid in the foam separates from it (Figure 18). After that the foam dries out. The result is that the foam becomes dry and stable faster.

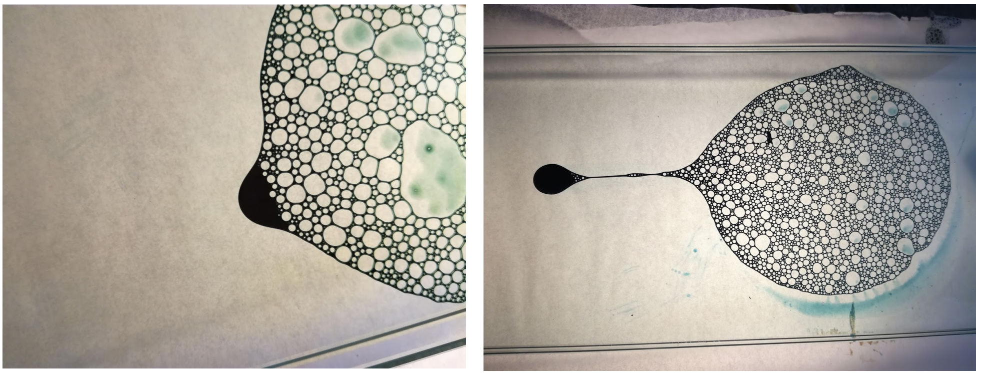


Figure 18. Different stages of the liquid separation phase

3.7. Observations on the behaviour of the foam at high inclinations

At high inclinations the liquid separation process appears again, but there is also a significant movement of the bubbles. Their velocities can be measured with Tracker. As we can see in Figure 19, they decrease approximately exponentially with time.

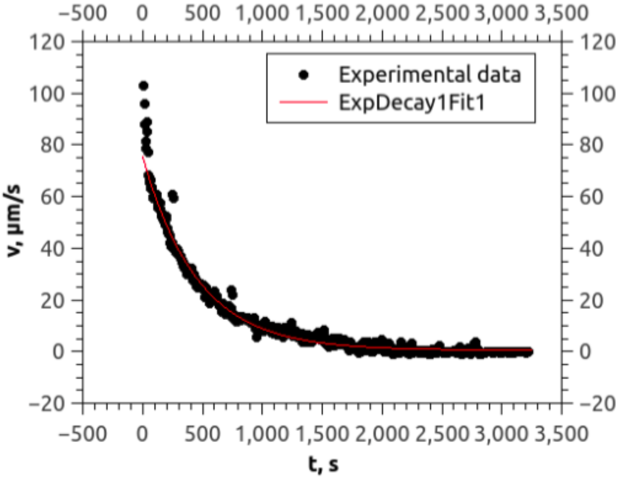


Figure 19. The velocity of bubbles along the long side of the glass sheets as function of time for inclination of 65°, w = 54% and d = 0.55 mm

4. Conclusion

This article has presented many fundamental properties of 2D soap foams, from Plateau's laws to their behaviour when inclined. Relevant parameters were varied and, where possible, their impact on the structure and evolution of a foam was estimated. While this study might not be perfect on all points, it could hopefully inspire and provide initial guidelines for future investigation in the field. Foams, and 2D liquid soap foams in particular, remain a mysterious and uninvestigated topic in science.

NOTES

1 For the sake of simplicity, the common abbreviation "ND" will often be used instead of "N-dimensional".

2 Here by area is meant the area of the visible polygonal (or roughly circular) figure (refer to Figure 1 and Figure 2), or roughly half of the whole surface area of the bubble.

3 We will call a foam stable if it doesn't evolve anymore.

4 Here small is an inclination between 0° (including) and 20°.

5 Preliminary experiments showed that after about 2 hours the evolution is too slow to observe.

REFERENCES

Mancini, M. (2005). *Structure and evolution of soap-like foams*: 1 – 2. Université de Cergy Pontoise.

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