https://doi.org/10.53656/nat2021-3.03

Experiments

PROBLEM OF THE 8-TH EXPERIMENTAL PHYSICS OLYMPIAD, SKOPJE, 8 MAY 2021 DETERMINATION OF PLANCK CONSTANT BY LED

Todor M. Mishonov[1)], Aleksander P. Petkov[1)], Matteo Andreoni[1)], Emil G. Petkov[1)], Albert M. Varonov[1)], Iglika M. Dimitrova[1), 2)], Leonora Velkoska[3)], Riste Popeski-Dimovski[3)]

[1)]Georgi Nadjakov Institute of Solid State Physics – BAS (Bulgaria)
[2)]University of Chemical Technology and Metallurgy (Bulgaria)
[3)] "Ss. Cyril and Methodius" University – Skopje (R. N. Macedonia)

Abstract. This is the problem of the 8th International Experimental Physics Olympiad (EPO8). The task of the EPO8 is to determine Planck constant $\hbar = h/2\pi$ using the given set-up with LED. If you have an idea how to do it, do it and send us the result; skip the reading of the detailed step by step instructions with increasing difficulties. We expect the participants to follow the suggested items – they are instructive for physics education in general. Only the reading of historical remarks given in the first section can be omitted during the Olympiad without loss of generality. Participants should try solving as much tasks as they can without paying attention to the age categories: give your best.

Keywords: measurement of Planck constant; light emitting diode (LED); compact disk (CD) diffraction grating; Experimental Physics Olympiad (EPO); current-voltage characteristics I – V curves

I. EPO – a historical perspective. Skip this section during the Olympiad

From its very beginning, the Experimental Physics Olympiad (EPO) is world-wide known; all Olympiad problems have been published in Internet (V. G. Yordanov et al. 2015; V. N. Gourev et al. 2015; S. G. Manolev at al. 2015; V. G. Yordanov et al. 2016; T. M. Mishonov et al. 2018; T. M. Mishonov et al. 2018; T. M. Mishonov et al. 2020) and from the very beginning there were 120 participants. In the last years high-school students from more than 10 countries participated and the distance between the most distant cities is more than 4 Mm.

Let us describe the main differences between EPO and other similar competitions.

– Each participant in EPO receives as a gift from the organizers the set-up, which one worked with. In such a way, after the Olympiad has finished, even

bad performed participant is able to repeat the experiment and reach the level of the champion. In this way, the Olympiad directly affects the teaching level in the whole world. After the end of the school year, the set-up remains in the school, where the participant has studied.

– Each of the problems is original and is connected to fundamental physics or the understanding of the operation of a technical patent.
– The Olympic idea is realized in EPO in its initial from and everyone willing to participate from around the world can do that. There is no limit in the participants' number. On the other hand, the similarity with other Olympiads is that the problems are direct illustration of the study material and alongside with other similar competitions mitigates the secondary education degradation, which is a world tendency.
– One and the same experimental set-up is given to all participants but the tasks are different for the different age groups, the same as the swimming pool water is equally wet for all age groups in a swimming competition.
– One of the most important goals of the Olympiad is the student to repeat the experiment at home and to analyze the theory necessary for the understanding. In this way any even badly performed motivated participant has the possibility to be introduced to the corresponding physics field, even though there is no physics classroom in his/her school, even though the physics education in his/her country to be deliberately destroyed.

We will briefly mention the problems of former 7 EPOs: 1) The setup of EPO1 (Yordanov et al. 2015) was actually a student version of the American patent for auto-zero and chopper stabilized direct current amplifiers. It was notable that many students were able to understand the operation of an American patent without special preparation (Goldberg & Jules 2015). 2) The problem of EPO2 (Gourev et al. 2015) was to measure Planck constant by diffraction of a LED light by a compact disk. 3) A contemporary realization of the assigned to NASA patent for the use of negative impedance converter for generation of voltage oscillations was the set-up of EPO3 (Manolev et al. 2015). 4) EPO4 (Yordanov et al. 2016) was devoted to the fundamental physics – to determine the speed of light by measuring electric and magnetic forces. The innovative element was the application of the catastrophe theory in the analysis of the stability of a pendulum. 5) The topic of the EPO5 (Mishonov et al. 2018) was to measure the Boltzmann constant k_B following the Einstein idea of study thermal fluctuations of electric voltage of a capacitor. 6) The EPO6 (Mishonov et al. 2018) problem can be considered as a continuation of the previous Olympiad. With a similar electronic circuit Schottky noise is measured and his idea for the determination of the electron charge is realized. 7) The EPO7 (Mishonov et al. 2020; Mishonov et al. 2020) problem was to measure a large inductance made by a general impedance converter by the Maxwell-Wien bridge. Each problem given at EPO can be considered as a dissertation in methodology of physics education.

In short, the established traditions are a balance between contemporary working technical inventions and fundamental physics.

The EPO problems are meant for high school and university students but are posed by teachers with co authorship with colleagues working in universities or scientific institutes. For colleagues interested in new author's problems for the needs of the contemporary physics education we share our experience in the description of the experimental set-ups described at a university level. These are for instance:

1) The determination of the Planck constant without light but only with electronic processes study (Damyanov et al. 2015); this set-up requires the usage of an oscilloscope but in some countries the oscilloscopes are available in the high school's physics labs and the price of the former is constantly going down.
2) The speed of the light without the usage of scales or high frequency equipment is another innovative set-up (Mishonov et al. 2017) for high school education. And the idea for this experiment is given by our teacher in electrodynamics Maxwell.
3) In the physics curriculum in all countries, it is mentioned that the temperature is a measure of average kinetic energy of the gas molecules but the Boltzmann constant kB that gives the relation between energy and temperature is not measured in high school and even rarely in the best universities. The experimental set-up for kB by the method proposed by Einstein (the EPO5 problem) is described (Mishonov et al. 2019) as a set-up for university school lab exercise in an impact factor journal. But what larger impact an experimental set-up that is used by high school students from Kazakhstan and Macedonia and the surrounding countries can have. More than 100 set-ups were distributed around the world.
4) Similar thoughts can be expressed for the electron charge qe. This fundamental constant is also mentioned in the high school education as a humanitarian incantation but is not measured. We broke this tradition and described an experimental set-up (from EPO6) in the European Journal of Physics (Mishonov et al. 2018). This set-up can be built for a week in every high school. The Schottky idea for determination of the electron charge by measuring voltage fluctuations is used. From the idea to the realization more than 100 years have passed and one of the reasons is that in many countries the largest enemy of the education is the ministry of education.

The development of some set-ups required additional study of circuits with operational amplifiers (OpAmps). This led to introduction of the master equation of OpAms applied to the stability of circuits with Negative Impedance Converter (NIC) (Mishonov et al. 2019), Generalized Impedance Converter (T. Mifune et al. 2021) and study of Probability Distribution Function (Mishonov et al. 2021) of the crossover frequency of OpAmps.

The mission of the physics teachers in the worldwide progress is evident – precisely our science reshaped the world in the last century. Successful innovative EPO set-ups after some update can be manufactured by companies specialized in production of educational equipment like TeachSpin[1)] and PHYWE[2)] for instance.

II. Description of the experimental set-up and the conditions for online participants

The set-up you received is represented in the Figure 1. Ones again we repeat the condition that the room in which you work can be darken by dense curtains, for example. The details which every participant has to ensure are depicted in Figure 2. In the next sections different tasks with increasing difficulty are described in the sections corresponding to every age category. The jury will look at the experimental data, tables and graphics. It is not necessary to write the humanitarian text between them, only mark the number of the corresponding task. We wish you success.



Figure 1. Details of the set-up given to the participants: 1 small plastic bag containing 8 light emitting diodes (LED), 1 paper clip and 2 small rubber rings, 1 potentiometer, 1 piece of black plastic straw, 3 metal binder clips, 1 battery holder for 3 AA-type batteries, 1 double sided adhesive tape, 1 compact disk (CD), 7 connecting crocodile cables, 1 torch with battery holder taken out, 5 bands of 6 differently colored sticky labels



Figure 2. Details which every participant has to ensure himself: computer with Internet connection, mobile phone, a camera or a scanner to create a PDF or picture file of your paper work during the Olympiad. Additionally: 1) 2 multimeters with original connecting cables, 2) 3 AA type batteries of 1.5 V, 3) 3 AAA type batteries of 1.5 V, 4) pen(s) and a plastic ruler, 5) 2 (or 1 if height of cube > 6 cm) paper cubes 8.8 cm 6) scissors, 7) at least 2 sheets of millimeter paper or squared paper sheets and 4 white sheets of ordinary A4 paper, 8) 2 sheets of A4 white paper with 240 g/m^2 or larger thickness, 9) calculator (not shown here and highly recommended :-), 10) pad for A4 paper (recommended), 11) pencil, rubber eraser, white band eraser (recommended), *) sandwiches and bottle of water (strongly recommended). 0) Nothing excessive on the cleaned working table and the room has to be darkened

III. Tasks S. Getting to know the voltage source

1. Turn on the multimeter as an Ohm-meter (W). With maximum accuracy measure the 2 soldered resistors to the triple AA battery holder and write down the results. What is the resistance r_+ and r_- of the resistors at the (+) and (–) ends of the 3 battery holder? Calculate $r = r_+ + r_-$.
2. What is the resistance R_{LR} between the left and right ends of the potentiometer?
3. Measure the resistance of the potentiometer between the middle terminal and one of the others. In what interval does the resistance change when you rotate its axis (R_{min}, R_{max})?
4. Turn on the multimeter as a DCV and measure the voltages of your 1.5 V AA batteries. Order them if they are different $E_1 \le E_2 \le E_3$.
5. Put the batteries at battery holder and measure the total electromotive force E. What is the accuracy of the relation $E = E_1 + E_2 + E_3$?
6. Calculate how many mV is the difference between ½ V and ⅓ V?

7. Using crocodile cables, battery holder, potentiometer and voltmeter connect the voltage source circuit depicted in Figure 3. Determine the the interval in which the voltage measured by the voltmeter changes U_V Î (0, U_{max}) when you rotate the axis of the potentiometer (ensure $U_V > 0$). You can use a piece of double side adhesive band to fix the potentiometer on the pad.

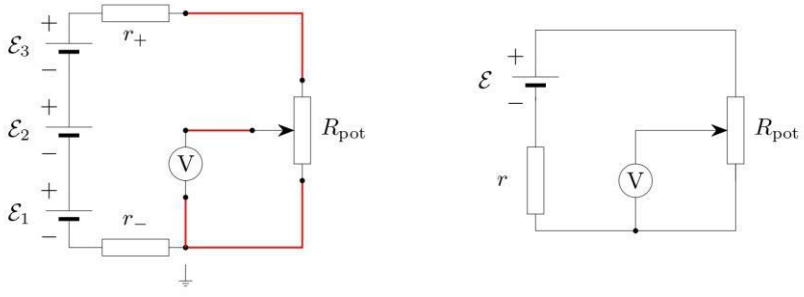


Figure 3. Left. Detailed circuit with: battery holder with 3 AA type batteries, two resistors, potentiometer, and voltmeter. Connecting 4 cables are given by red color and crocodiles clips are marked by points (dots). Right. Schematic representation of the same circuit: voltage source with electromotive voltage $E = E_1 + E_2 + E_3$ and sequential internal resistor $r = r_- + r_+$ corresponding to the battery holder with the 3 AA type batteries inserted in, potentiometer with resistance R_{pot}, and voltmeter

8. Using the circuit from Figure 3, connect two additional crocodile cables to the middle terminal of the potentiometer and the ground point of the circuit, leaving the other connectors of the two cables open. In such a way the voltmeter shows the voltage applied to the LED $U = U_V$.
9. Note that light-emitting diodes (LED) have terminals with different lengths; *the longer (anode) has to be connected to positive voltage*. Connect the open ends of the two cables to one of the LEDs, following Figure 4. Do you see the similarities and difference between in the connections in Figure 3 and Figure 4?

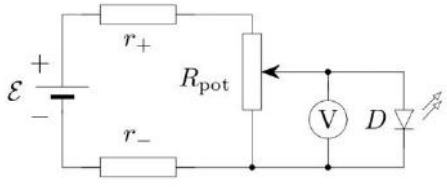


Figure 4. LED connected to the voltage source from Figure 3, circuit for measuring the voltage U^* at which the LED emits the weakest evanescent light

10. Turn on the voltage source until the connected LED begins emitting clear visible light. Make a table with 3 columns, column 1 is the sequential number of the LED, column 2 is its color, column 3 is U^* the smallest voltage at which the LED emits light. From now on you have to keep the enumeration and order of the LEDs, you can use the colored sticky labels for that but make sure you can easily remove the labels if necessary.
11. Now it is good to darken the room. Put the connected LED into the one end of the plastic straw. Measure U^* by carefully rotating the handle of the potentiometer and visually observing with your eye through the other end of the plastic straw when the LED begins emitting light (the plastic straw is a waveguide for the LED light). We recommend at least 5 different measurements and taking the median (or middle) value for U^*. In this way determine U^* for all available and working LEDs and fill column 3 of the table. The human eye is a very good sensor however the results are subjective.

IV. Tasks M. Diffraction of LED light

The purpose of this set of tasks is to measure the the wavelength l and the frequency of the light $n = c/l$ of every LED; where c » 300 Mm/s is the speed of light. For diffraction grating we will use Compact Disk (CD) with constant d = 1600 nm.

12. Cut a 8.8 cm square piece of cardboard or thicker paper. Using the double adhesive tape carefully stick all measured LEDs in item 11 on this square piece as shown in Figure 5 and enumerate them according to item 10. It is better the ends of the diodes to be bent vertically in order to be connected by crocodile cables easier. For further convenience, try using one and the same orientation of the terminals, or mark with (+) on the cardboard the longer terminal. If you cannot accomplish this item, you can still solve the whole problem, therefore proceed to the next tasks.
13. Now you need a diffraction grating. In order to obtain it you have to remove the metallic foil layer from the CD as it is described in Figure 6.

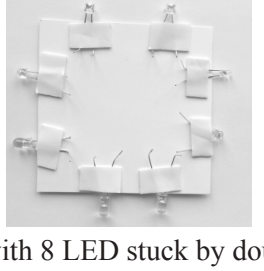


Figure 5. Square piece with 8 LED stuck by double sided adhesive tape. Enumerate each LED and write a small (+) sign close to the number to mark the anode of LED. The terminals are bent in vertical direction in order to be bitten by crocodiles :-)



Figure 6. The technology of peeling the foil of the compact disk: 1) The foil is scratched radially with the paper clip. 2) A piece of the adhesive tape is stuck on the scratch. 3) Then the piece is pulled, and the foil stuck on the tape is removed from the CD. 4) Repeat steps 2) and 3) above (or near) the edges of the remaining foil until about half part of the disc is peeled from the foil as a half Moon at the right. You can use both sides of the double adhesive band

14. Put the 3 AAA type batteries in the torch battery holder, assemble the torch, turn it on and focus it at infinity (make the beam as narrow as possible). You have almost parallel beam of white light. Using the different halves of the disk, you can observe a spectrum of the torch white LED at reflection and transmission. In order to observe spectrum at transmission, the light from the torch has to pass through the peeled half of the disc and fall on a blank sheet of paper. The sheet of paper and the CD have to be parallel, and we recommend the distance between them *D* to be approximately 15 cm. Try obtaining central maximum and two horizontal spectra with the torch; this is the initial exercise to the more difficult task with the set of LEDs.
15. Take out the lens from the torch as shown in Figure 7.



Figure 7. Torch with lens: carefully unscrew the part holding the lens to the torch, disassemble it and take out the lens

16. Fix the lens at the top of 1 metal binder clip with rubber rings as it is shown in Figure 8 and Figure 9. Stick (highly recommended) the binder clip with the lens on the pad (or table) with a piece of double-sided adhesive tape.
17. Use the paper cubes (or cube) and adjust the height approximately equal to the height of the lens on the binder clip.

18. Place the 8.8 cm squared piece with LEDs made in item 12 on the cubes, bring them together near the lens, so that one of the LEDs to face the lens and be at close distance behind it as shown in both Figure 8 and Figure 9. You can use books instead of the paper cubes, but height change is more difficult.
19. Put the CD in the second metal binder clip and place it after the lens as shown in both Figure 8 and Figure 9. The binder clip with the lens must be between the diodes and the binder clip with the CD.
20. Put the cardboard box with a normal sheet of paper or the thicker sheet of paper in the third metal binder clip. This is your screen.
21. Carefully arrange the diodes, binder clip with lens, binder clips with CD and screen as shown in both Figure 8 and Figure 9. Screen, CD and lens should be parallel to each other and perpendicular to the direction of propagation of light. It is best to use the periphery of the CD for the diffraction region.

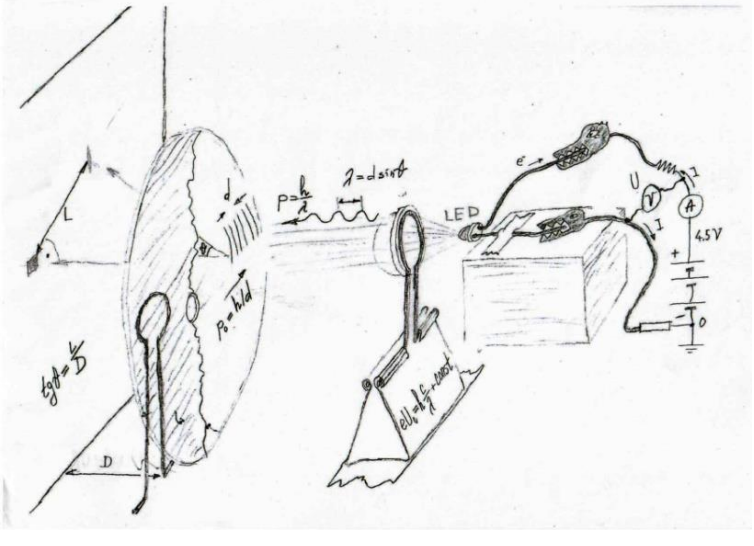


Figure 8. Scheme of the setup shown on Figure 9. The LED is connected with wires to the voltage supply source using crocodile clips. A photon from the collimated light has momentum $p = h/l$, which is perpendicular to the CD. The photon obtains momentum $p_0 = h/d$ in direction perpendicular to the direction of the tracks of the CD, and for the diffraction angle q we have $\tan q = p_0/p$. D is the distance between the screen and the CD, and L is the distance between the central and the first diffraction maximum; $\tan q = L/D$. The binder clips, which hold the lens and the CD, are drawn schematically, too

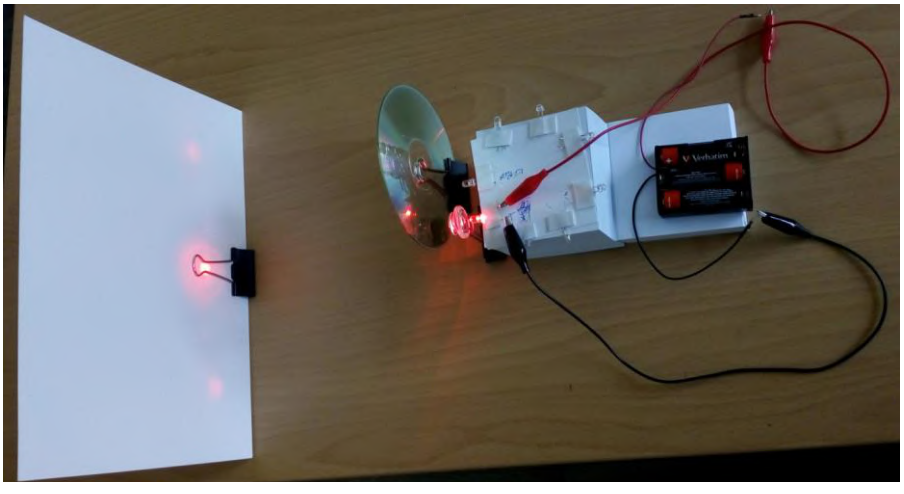


Figure 9. Experimental set-up for measuring the wavelength of LED light. The red LED is connected with the connecting cables to the voltage supply source. The light is collimated with the lens then it passes through the tracks of the transparent compact disk and one can see a bright image of the LED on the screen. Together with the central maximum two pale diffraction maximums can be observed on the screen. Diffraction angles are different for the different colors from the set. The wavelength l of the light can be determined by measuring the diffraction angles q

22. Use the voltage source and apply E to one of the red LED and place it behind the lens. Make additional adjustments in the arrangement until you see the bright central maximum and pale diffraction maximums on your screen as in Figure 9. The focus distance is very small and putting the LED close to the lens you can see the real image of the lighting part of the led on the screen. The method for adjusting of the set-up is individual, we recommend all details to be on the paper pad where with pen you can mark the place of every detail which can be easily returned in the same place after removal.
23. Try to make fine tuning of the set-up slightly rotating or moving the screen and/or CD. Additionally, adjust the height of the square piece with LEDs if necessary. The distances between central maximum and left L_l and right one L_r should be approximately equal and approximately on a horizontal line. Experimentally the mean value $L = ½(L_l + L_r)$ can be determined as half of the distance between the diffraction maximums.
24. For all of LEDs you have to measure L and the distance D between the screen and CD, which is better to be kept constant for each LED with the

adhesive tape. Then we can calculate sine of the diffraction angle q = arctan (L/D) using the distances $\sin\theta = L/\sqrt{L^2 + D^2} = \sin\left(\arctan(L/D)\right)$. The wavelength then is given by the well-known formula

$$\lambda = d\sin\theta \tag{1}$$

Now you can calculate the frequency $n = c/l = 1/T$ and the period T of the oscillations.

25 The determination of the frequency of light of all LEDs is the main goal of the optical part of the task. We recommend the measured and calculated data for all diodes to be represented in Table I.

Table I: Table for arranging the measured results from the LED diffraction patterns. We recommend not to draw the vertical lines of the table since they quite often make it unusable.

#	color	D [mm]	L [mm]	sinθ	θ [deg]	λ [nm]	ν [THz]	q_eU^* [10^{-19} J]
1	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...

26. Now you can address the main goal of the Olympiad – the determination of Planck constant h. Using the electron charge $q_e = 1.60.10^{-19}$ C complete the last column of Table I by calculating the energies q_eU^* for all LEDs, using the results from item 11. Those energies are related to photon energies by the equation

$$h\nu = q_eU^* + A \tag{2}$$

Our estimation for Planck constant h is based on the approximation that the parameter A is weakly dependent of the chemical compound of the LED semiconductor therefore is almost constant.

27. Carefully consider the range of variables, the used units and their powers. Mark in a graphic plot of millimeter paper: (x) in the abscissa the frequencies of the LED and (y) along the ordinate axis the energies q_eU^*. You can use, for example, THz and 10^{-19} J. In this (n, q_eU^*) plot places the points corresponding to all measured LEDs. For each point write down the number according to the table in item 11.

28. Perform linear regression of these points (draw a straight line which passes closest to all points according to you). It is allowed to ignore a point if it deviates too much from the others and if you do it, write it down explicitly. If you perform the linear regression with a programmed calculator, addition-

ally write down the correlation coefficient of the linear regression.

29. On the straight line choose two points (A) and (B) and write down their coordinates: energies E_A and E_B and frequencies n_A and n_B. Calculate the differences $\Delta E = E_B - E_A$ and $\Delta n = n_B - n_A$.
30. Calculate the ratio $h_{LED} = \Delta E/\Delta n$. This is your evaluation of the Planck constant. At correct solution of the problem, you will obtain at least the correct order.
31. Calculate the ratio $X = h_{LED}/h$ between the measured by you with the present set-up value and the well-known value $h = 6.62.10^{-34}$ J/Hz. Calculate also $e = 100(X - 1)$, which is roughly speaking the error in percents. For some purposes is more convenient to use frequency $w = 2\pi n$. In this case $\hbar = h/2\pi = 1.054.10^{-34}$ Js is introduced and $hn = \hbar w$.

V. Tasks L. Current voltage characteristics of LED

32. Choose one of your multimeters to work as an ammeter, and the other to work as a voltmeter. Then measure the internal resistance of the ammeter R_A at its smallest current range.
33. To study the current voltage characteristics, $I(U)$ or I-V curves of the LEDs, connect the circuit from Figure 10.

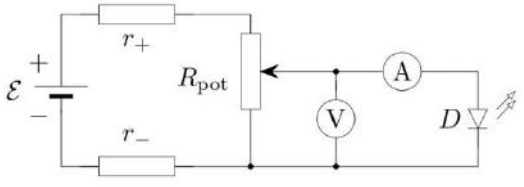


Figure 10. Circuit for study current-voltage characteristics. Adjustable voltage regulator with potentiometer for voltage supplying to the LEDs and for investigation of $I(U)$. The voltage from the battery holder is applied to the potentiometer. The LED is connected to the middle terminal and to one of the other terminals of the potentiometer. The ammeter is connected in series with the LED and the voltmeter is connected in parallel

34. We recommend to start with a red LED and sequentially study the others from the set. The ammeter measures the current I_A through the LED I, i.e. $I = I_A$. The voltmeter shows voltage U_V and the voltage at the LED is $U = U_V - R_A I$.
35 .In this task we focus on small currents in the range I Î (0.1, 10) mA. Rotating the axis of the potentiometer you have to look at the ammeter. For each LED you have to record less than 10 current values below and above 1 mA. Observe the voltage values also, if necessary, adjust the multi-meter range.

Arrange your data in a table similar to Table II.

Table II: Tabulated current voltage characteristics per LED, for each studied LED you need a separate table.

UV [V]	I [mA]
...	...
...	...

36. Draw the I – V curve on a millimeter or squared paper. Mark in a vertical arrow U^* at the abscissa U [V]. Use the approximation of ideal ammeter $R_A \approx 0$ when $U \gg U_V$.
37. Try to invent a method for determination of U^* analyzing only the I-V curves at small currents and illustrate it graphically.

VI. Tasks XL. University students

38. Perform the linear regression of the experimental points in the plot $(n, q_e U^*)$ with a computer (if available) and calculate more precisely h_{LED} and the correlation coefficient. It is allowed to omit an experimental point with explanation of the reasons.
39. If a point has been omitted, compare h_{LED} with the one obtained with all points. Does the omission of a point decreases e and increases the correlation coefficient?
40. Perform the order estimation of the errors in measurement of U^*. You have at least 5 measurements.
41. Evaluate the error in the measurement of the diffraction angle q and relate to it the wavelength l.
42. Evaluate A in eV both from item 38 (if available) and from item 28.
43. Consider what gives larger uncertainty for the final result inaccuracies in ΔU^* and Δq or the approximation that the band gap parameter – A is the same constant for all LEDs.
44. The use of human eye inserts subjective uncertainties. Invent a numerical method to determine the voltage U^* analyzing I-V curves (current-voltage characteristics). Apply polynomial regression in some interval and fit to exponent in another. For example, use quadratic polynomial (parabolic) fit

$$I = aU^2 + bU + c = a\left(U - \tilde{U}\right)^2 + \left(c - b^2/4a\right) \qquad (3)$$

for currents in the interval $I \in (1, 10)$ mA and calculate the position of the minimum $\tilde{U} = -b/2a$. Calculate $|\tilde{U} - U^*|/U^*$ for the LEDs you study.

45. Compare the obtained $\tilde{U}$ from the previous task and U^* from item 11 for all LEDs. Present the errors between the two types of measurements in percent.

VII. Problems for Further Work After the Olympiad

46. The price of LEDs is very low. You can order LEDs with marked by the manufacturer wavelength and compare with your determination of the wavelength. We expect 5-10% accuracy. If you find a good LED, please let us know at epo@bgphysics.eu.
47. What accuracy of determination of the wavelength l can be reached if you use lasers?
48. To Table II add 3 additional columns to the right: the correction $R_A I$, the corrected LED voltage $U = U_V - R_A I$, and the LED resistance $R(U)$ º U/I in MW as is demonstrated in Table III. Compare the minimal resistance from the table R_{min} with the resistance of the voltmeter R_V in the used range.

Table III: Complete current voltage characteristics in which finite internal resistance of the ammeter is taken into account.

U_V [V]	I [μA]	$R_A I$ [V]	U [V]	R [MΩ]
...	...	...	...	...
...	...	...	...	...

49. If the current through the LED I is large enough its resistance U/I is very small and much smaller than the voltmeter internal resistance. In this case it is necessary to use the connection depicted at Figure 11.

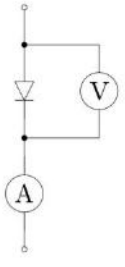


Figure 11. Connection for high current and low LED resistance. In this case $U = U_V$ and the current through the LED $I = I_A - U/R_V$. When the voltmeter internal resistance is high enough $1/R_V \approx 0$, the corrections to the current U/R_V are negligible

50. If laser light falls on a vacuum lamp you can reach the accuracy of the first measurement of the Planck constant h by Millikan (R. Millikan 1916). Check what the accuracy of the determination of h is in the contemporary university labs.

51. Send to the address of the Olympiad epo@bgphysics.eu your responses, recommendations, criticism and suggestions, which you think would help the EPO9 organizers. Negative impressions are also welcome and will be taken into account.
52. Re-derive and program the solution equations for the parabolic fit of a set of experimental data $I = aU^2 + bU + c$. Those are conditions for the minimization of the function

$$f(a,b,c) \equiv \sum_{i=1}^{N} \left(aU_i^2 + bU_i + c - I_i \right)^2 ,$$

where summation is on the all N points. The conditions for minimum require the annulation of the first derivatives $\partial_a f = 0$, $\partial_b f = 0$, and $\partial_c f = 0$, read

$$\left\langle U^{2+m} \right\rangle a + \left\langle U^{1+m} \right\rangle b + \left\langle U^{0+m} \right\rangle c = \left\langle U^m I \right\rangle ,$$

where brackets denote averaging, for example

$$\left\langle U^m I \right\rangle = \frac{1}{N} \sum_{i=1}^{N} U_i^m I_i , \qquad \left\langle U^k \right\rangle = \frac{1}{N} \sum_{i=1}^{N} U_i^k , \qquad k = 0,1,2,3,4 .$$

In such a way we have a system of 3 linear equations, m = 0, 1, 2, for the parameters a, b and c which solutions can be easily programmed and of course it is implemented in many software programs addressed for users.

53. Draw the parabolic fit on the plot with experimental data (U_i, I_i) and draw a small circle at the minimum of the parabola $\tilde{U} = - b/2a$. Try to invent a better method for determination of the U^* used for determination of Planck constant.
54 Most easily multimeters can be burnt when are used as ammeters. That is why this task can be done after the Olympiad. Every time start with the maximal fused current range. Switch one of multimeters as ammeter and measure the maximal current I_{max} which can give our voltage source. Can we evaluate the internal resistance of the batteries r_{bat} using $I_{max} = E/(r + R_A + 3\, r_{bat})$?

VIII. Epilogue

The complete solution of the problem is given in the appendix of the arXiv vertion (Mishonov et al. 2021) of the present article.

EPO8 is held in a very difficult for the whole world conditions. Approximately half of the participants are on-line. The organizers of EPO8 would like to thank everyone who helped in the preparation of this wonderful competition especially the president of the Society of Physicists of Macedonia, Prof. Lambe Barandovski for ensuring on-spot participation in Skopje. We are waiting you at EPO9; next year same time.

NOTES

1. Teachspin Inc., https://www.teachspin.com/.
2. PHYWE, https://www.phywe.com/.

REFERENCES

V. G. Yordanov, P. V. Peshev, S. G. Manolev, T. M. Mishonov, 2015. Charging of capacitors with double switch. The principle of operation of auto-zero and chopper-stabilized DC amplifiers. *Bulgarian Physics Education,* **3**(1 – 2), 33 – 54, (in Bulgarian). Available on: arXiv:1511.04328 [physics.ed-ph].

V. N. Gourev, S. G. Manolev, V. G. Yordanov & T. M. Mishonov, 2015. Measuring Plank constant with colour LEDs and compact disk. *Bulgarian Physics Education,* **3**(1 – 2), 55 – 86, (in Bulgarian). Available on: arXiv:1602.06114 [physics.ed-ph].

S. G. Manolev, V. G. Yordanov, N. N. Tomchev & T. M. Mishonov, 2015. Volt-Ampere characteristic of "black box" with a negative resistance. *Bulgarian Physics Education,* **3**(3-4), 109-151, (in Bulgarian). Available on: arXiv:1602.08090 [physics.ed-ph].

V. G. Yordanov, V. N. Gourev, S. G. Manolev, A. M. Varonov & T. M. Mishonov, 2016. Measuring the speed of light with electric and magnetic pendulum. *Bulgarian Physics Education,* **4**(1-2), 109-151, (in Bulgarian). Available on: arXiv:1605.00493 [physics.ed-ph].

T. M. Mishonov, E. G. Petkov, A. A. Stefanov, A. P. Petkov, I. M. Dimitrova, S. G. Manolev, S. I. Ilieva & A. M. Varonov, 2018. Measurement of the Boltzmann constant by Einstein. Problem of the 5-th Experimental Physics Olympiad. Sofia 9 December 2017. *arXiv:1801.00022v4 [physics.ed-ph]*.

T. M. Mishonov, E. G. Petkov, A. A. Stefanov, A. P. Petkov, V. I. Danchev, Z. O. Abdrahim, Z. D. Dimitrov, I. M. Dimitrova, R. Popeski-Dimovski, M. Poposka, S. Nikolić, S. Mitić, R. Rosenauer, F. Schwarzfischer, V. N. Gourev, V. G. Yordanov & A. M. Varonov, 2018. Measurement of the electron charge q_e using Schottky noise. Problem of the 6-th Experimental Physics Olympiad. Sofia 8 December 2018. *Eur. J. Phys.* **39** 065202. Available on: arXiv:1703.05224v2 [physics.ed-ph].

T. M. Mishonov, R. Popeski-Dimovsk, L. Velkoska, I. M. Dimitrova, V. N. Gourev, A. P. Petkov, E. G. Petkov & A. M. Varonov, 2020. The Day of the Inductance. Problem of the 7-th Experimental Physics Olympiad, Skopje, 7 December 2019. *Chemistry: Bulgarian Journal of Science Education,* **29**(3). Available on: arXiv:1912.07368 [physics.ed-ph].

T. M. Mishonov, R. Popeski-Dimovski, L. Velkoska, I. M. Dimitrova, V. N. Gourev, A. P. Petkov, E. G. Petkov & A. M. Varonov, 2020. The Day of the Inductance. Problem of the 7th Experimental Physics Olympiad, Skopje, 7 December 2019, *Chemistry: Bulgarian Journal of Science Education.* **29**(3), 380 – 397.

E. A. Goldberg & L. Jules, 2015. Stabilized direct current amplifier. *Patent US2684999A.*

D. S. Damyanov, I. N. Pavlova, S. I. Ilieva, V. N. Gourev, V. G. Yordanov & T. M. Mishonov, 2015. Planck's constant measurement by Landauer quantization for student laboratories. *Eur. J. Phys.* **36**, 055047.

T. M. Mishonov, A. M. Varonov, D. D. Maksimovski, S. G. Manolev, V. N. Gourev & V. G. Yordanov, 2017. An undergraduate laboratory experiment for measuring e_0, m_0 and speed of light c with do-it-yourself catastrophe machines: electrostatic and magnetostatic pendula. *Eur. J. Phys.* **38**, 025203.

T. M. Mishonov, V. N. Gourev, I. M. Dimitrova, N. S. Serafimov, A. A. Stefanov, E. G. Petkov and A. M. Varonov, 2019. Determination of the Boltzmann constant by the equipartition theorem for capacitors. *Eur. J. Phys.* **40**, 035102.

T. M. Mishonov, E. G. Petkov, N. Zh. Mihailova, A. A. Stefanov, I. M. Dimitrova, V. N. Gourev, N. S. Serafimov, V. I. Danchev & A. M. Varonov, 2018. Simple do-it-yourself experimental set-up for electron charge q_e measurement. *Eur. J. Phys.* **39**, 065202.

T. M. Mishonov, V. I. Danchev, E. G. Petkov, V. N. Gourev, I. M. Dimitrova, N. S. Serafimov, A. A. Stefanov & A. M. Varonov, 2019. Master equation for operational amplifiers: stability of negative differential converters, crossover frequency and pass-bandwidth. *J. Phys. Commun.* **3**, 035004.

T. Mifune, T. M. Mishonov, N. S. Serafimov, I. M. Dimitrova, R. Popeski-Dimovski, L. Velkoska, E. G. Petkov, A. M. Varonov, A. Barone, 2021. Tunable high-Q resonator by general impedance converter. *Rev. Sci. Instrum.* **92**, 025123.

T. M. Mishonov, E. G. Petkov, I. M. Dimitrova, N. S. Serafimov and A. M. Varonov, 2021. Probability distribution function of crossover frequency of operational amplifiers. *Measurement* **179**, 109509.

R. Millikan, 1916. A Direct Photoelectric Determination of Planck's *h. Phys. Rev.* **7**(3), 355 – 388.

T. M. Mishonov, A. P. Petkov, M. Andreoni, E. G. Petkov, A. M. Varonov, I. M. Dimitrova, L. Velkoska & R. Popeski-Dimovski, 2021. Problem of the 8th Experimental Physics Olympiad, Skopje, 8 May 2021 Determination of Planck constant by LED. *arXiv:2106.01337v1 [physics.ed-ph]*.

✉ **`Todor M. Mishonov (corresponding author)**
ORCID iD: 0000-0002-6945-9632
Georgi Nadjakov Institute of Solid State Physics
Bulgarian Academy of Sciences
Faculty of Physics
University of Sofia
5, James Bourchier Blvd.
1164 Sofia, Bulgaria
E-mail: epo@bgphysics.eu

Aleksander P. Petkov
Georgi Nadjakov Institute of Solid State Physics
Bulgarian Academy of Sciences
72, Tzarigradsko Chaussee Blvd.
1784 Sofia, Bulgaria

Matteo Andreoni
Georgi Nadjakov Institute of Solid State Physics
Bulgarian Academy of Sciences
72, Tzarigradsko Chaussee Blvd.

Emil G. Petkov
Georgi Nadjakov Institute of Solid State Physics
Bulgarian Academy of Sciences
72, Tzarigradsko Chaussee Blvd.

Albert Varonov (corresponding author)
ORCID iD: 0000-0001-5394-3027
Georgi Nadjakov Institute of Solid State Physics
Bulgarian Academy of Sciences
72, Tzarigradsko Chaussee blvd.
1784 Sofia, Bulgaria
E-mail: epo@bgphysics.eu

Iglika M. Dimitrova
Georgi Nadjakov Institute of Solid State Physics
Bulgarian Academy of Sciences
Faculty of Chemical Technologies
Department of Physical Chemistry
University of Chemical Technology and Metallurgy
8, Kliment Ohridski Blvd.
1756 Sofia, Bulgaria

Leonora Velkoska
Faculty of Natural Sciences and Mathematics
Institute of Physics
"Ss. Cyril and Methodius" University
Skopje, R. N. Macedonia

Riste Popeski-Dimovski
Faculty of Natural Sciences and Mathematics
Institute of Physics
"Ss. Cyril and Methodius" University
Skopje, R. N. Macedonia
E-mail: ristepd@gmail.com