Математика
и информатика

Volume 65,
Number 2, 2022

Mathematics
and Informatics

https://doi.org/10.53656/math2022-2-3-pro

Educational Issues
Въпроси на преподаването

# PROBLEMS OF CASCADE TYPE AND THEIR USE FOR ASSESSMENT OF STUDENTS' ACADEMIC ACHIEVEMENTS IN MATHEMATICS

**Prof. Dr. Shkolnyi Oleksandr[1)], Prof. Shvets Vasyl[1)], Dr. Akiri Ion[2)]**
*[1)]National Pedagogocal Dragomanov University – Kyiv (Ukraine)*
*[2)]Institute of Pedagogical Sciences – Chisinau (Moldova)*

**Abstract.** Mathematical problems that contain a common condition and an ordered list of tasks are traditionally called cascade problems. Problems of this type are extremely popular during teaching of mathematics. At the same time, they can be used both in the formation of knowledge, skills and abilities (competencies) of students, and to assess their academic achievements. In the paper we give a thorough theoretical analysis of possible types of cascade problems, describe methodology and provide examples of their applications on different stages of studying mathematics at school, in particular, during the final attestation of graduates. According to our statistical survey of mathematics teachers in Ukraine and Republic of Moldova, we clarify their attitude to problems of cascade type, as well as the advantages and disadvantages of using such problems. It is shown that the use of cascading problems in the school course of mathematics contributes to the adequate formation of important competencies of students in their learning process.

*Keywords:* cascade problems; teaching mathematics; assessment of students' academic achievements; competence approach in teaching mathematics; development of students' thinking

**Introduction.** One of the important directions of modern pedagogical researches is to ensure the quality of the so-called "high stake testing" (see, for example, (Au 2011), (Berliner 2011), (Lingard et al. 2013)). These in Ukraine include the External Independent Assessment of graduates' academic achievements (EIA) and the State Final Attestation of students (SFA). In the United States, such tests include the SAT (Scholastic Assessment Test) and ACT (American College Test), in Russia – the Unified State Examination (USE), in the Republic of Moldova – the State Undergraduate Examination, in Germany – the final test (Abitur) and more. Detailed information on such testing in different countries can be read in monographs (Karp 2020) and (Shkolnyi 2015). In this article, we focus more on ensuring the proper quality of final and entrance standardized tests in Ukraine and Republic of Moldova.

Currently, the EIA in Ukraine is the only tool for competitive selection for admission to Ukrainian universities, as well as for the final assessment of the main learning outcomes of the secondary school course. According to the decision of the Ministry of Education and Science of Ukraine (MES of Ukraine 2019) from the 2020/2021 academic year, the test of external examination in mathematics was to become mandatory for the SFA. However, due to the pandemic situation, this obligation was temporarily postponed, and Ukrainian graduates have been exempted from passing the SFA in all subjects for the last two years (MES of Ukraine 2021). However, this does not mean that attention to this type of testing in Ukraine has become less.

It should be noted that even the very existence of EIA is now perceived by Ukrainian society ambiguously. In addition to positive reviews, which are much more (Rating Group Ukraine 2018), there are also negative (Cabinet of Ministers of Ukraine 2019). Therefore, it is clear that the existing system of independent evaluation needs further improvement and development. The history of the introduction and formation of external evaluation in Ukraine can be traced to sources (Dvoretska 2015), (Liashenko & Rakov 2008), (Shvets et. al. 2020) and others.

It is quite difficult to establish the main reason for the introduction of an independent testing system. On the one hand, the level of corruption during the admission campaign to Ukrainian universities at the end of the last century was so high that the public demanded adequate reactions from the state to overcome it (Protasova et. al. 2012). The problem of biased assessment of student achievement and corruption during the final certification exams was also a matter of considerable concern not only to education officials, but also to a large number of parents. The introduction of EIA was a significant step towards overcoming both of these negatives.

It should be noted that the development of test technologies in Ukraine was funded by the Institute of Open Society of J. Soros and the American Council under the USETI program (see (McLaughlin & Webber 2013) and (Liashenko & Rakov 2008)). This contributed to the implementation of Ukraine's course to harmonize its own educational system with European and world educational traditions, to ensure the quality of assessment of students' achievement on the level of world standards. Therefore, the introduction of EIA took place in stages and using the experience of other countries. Experts from the USA, Poland, Finland, Lithuania, Georgia, Austria and Switzerland joined. However, as the general approaches to high stake testing, for example, in Finland and the United States, differ significantly, it cannot be said that the current system of independent evaluation in Ukraine completely replicates a similar system of standardized testing in some of these countries, but combines systems mentioned above. For this reason, the current EIA test in mathematics now includes both popular in the American tests multiple choice questions and problems with full explanation spread in Finland, the leader of world comparative educational programs like PISA and TIMMS (Sahlberg 2015).

In the Republic of Moldova, the State Exams for the Bachelor's Degree are analogous to the Ukrainian EIA on the lyceum level (12th grade). These exams are also the only means of competitive selection during the entrance examinations to the universities of the Republic of Moldova and to the universities of Europe. A big problem during the Exams is the creation of examination tests in the context of the formation of competencies (see (Parliament of Moldova Republic2019) and (Ministry of Education of Moldova Republic 2021)).

In the Republic of Moldova, as a result of the education reform in 1998 – 2000, it was decided to combine the systems of final examinations in lyceums (12th grade) with entrance exams to universities. Admission to the university is based on a competition of grades obtained by secondary school students during the bachelor's exams. To this end, a system of state examinations for bachelor's degrees was introduced, by analogy with France and Romania (see (Parliament of Moldova Republic 2011) and (Ministry of Education of France 2000)). The exams in the Republic of Moldova were conducted and now are conducted on the base of exam tests passing.

Every year (in 2019 due to the pandemic situation, graduates were exempted from final exams), in June, graduates of lyceums of the Republic of Moldova (12th grade) take an examination session. Mathematics is a compulsory exam for the Real Profile and an elective exam for the Humanities. The exams are written and the students take the test in 180 minutes. The tests differ in the levels of difficulty for the respective profiles. Test tasks are standardized and aimed on assessing mathematical competencies. Mathematics tests mainly contain tasks with full explanation (Ministry of Education of the Republic of Moldova 2021).

As we can see, in the system of standardized testing in both Ukraine and the Republic of Moldova there are test tasks with a full explanation. Their importance is difficult to overestimate, because they are designed to test the ability of graduates to describe solutions, justify their actions, prove statements, make logical reasoning. Traditionally, solving such tasks is quite cumbersome, multi-step, and therefore, their assessment requires consideration of many factors. In Ukraine, a separate evaluation scheme is created for each of these tasks, where clearly prescribes for which stages of the solution the appropriate number of points is accrued. Naturally, for the convenience of creating such evaluation schemes, each task is explicitly divided into stages, for the implementation of each of which there are one, two or more points. For example, in the 2021 pilot EIA math test, one of the tasks was as follows.

*Task # 31* In a right quadrangular pyramid $SABCD$ the flat angle at the vertex $S$ of the pyramid is equal to $\beta$. The length of the apothem (slant height) of the pyramid is 6.

1. Construct an image of a given pyramid and mark the angle $\beta$.
2. Express the length of the side of the base of the pyramid $SABCD$.
3. Find the volume of the pyramid $SABCD$.

Solving of the problem involves the consistent accomplishing of items 1, 2, 3, and in each subsequent item it is needed to use the results of the previous one. Similar problems in the methodological literature are called cascade tasks (cascade problems). Let's dwell on them in more detail.

**Definition of the cascade problem. Examples of the cascade problems.** *Cascade problem* is a problem that contains common condition and an ordered list of tasks (subproblems) that should be accomplish to complete it. This list of subproblems can include both *sequential execution* (using the results of previous subproblems) and *parallel execution* (when the result of each individual subproblem does not affect the execution of other subproblems). It is possible to combine both approaches, when some subproblems require the results of others, and some do not. It is clear that the cascade task is not simple, but complex.

For example, in the aforementioned task # 31 of the Ukrainian pilot EIA math test in 2021 implemented the sequential method of forming subproblems of the cascade problem. Let's give examples of other types of cascade problems that were used for the final attestation of graduates in three European countries – Ukraine, the Republic of Moldova and Germany.

*Example # 1* (The Republic of Moldova 2010). Let $f(x) = x^2(x+1)(x-2)$.

1. Find the extremum points of the function $f$.
2. Write an equation whose roots are the numbers opposite to the values of $x$ obtained in item 1.
3. Find the inverse derivative function associated with the equation obtained in the item 2.
4. Calculate the integral $\int_0^2 g(x)dx$, where $g(x)$ is the inverse derivativ function found in the item 3.
5. Find the length of the edge of the cube, the total surface area of which is equal to the numerical value (in square units) obtained in the item 4.
6. Calculate the volume of the regular tetrahedron whose edge is equal to the edge of the cube in the item 5.

Proposed cascade problem makes it possible to simultaneously assess students' knowledge of three sections of mathematics – elements of mathematical analysis, algebra and geometry. The problem also contains task 2 for the development of creativity – students should make equations based on the given data. The structure of this task is such that it involves the sequent execution of all subproblems. Hereinafter, we will call such problems *cascade problems of linear type*.

*Example # 2* (Ukraine 2021). Define the function $y = x^3 - 3x$.

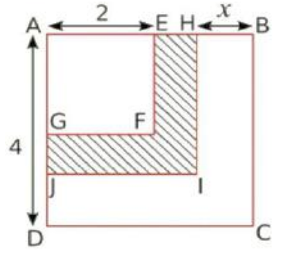
1. For given in the table values of argument x find respective values y

| *x* | *y* |
|---|---|
| 0 | |
| –1 | |
| 2 | |

2. Find and write the coordinates of intersection of the graph of function $y = x^3 - 3x$ with the axis *x* .
3. Find the derivative f ' (x) of the function $f(x) = x^3 - 3x$.
4. Determine the zeros of the function f ' (x).
5. Find the intervals of increase and decrease, points of extemums of the function *f*(*x*).
6. Draw the sketch graph of the function *f* .

The proposed task covers a range of different tasks related to the topic "Function" and allows teachers to check how deeply the student has mastered it. All subtasks of the problem are purely reproductive and aimed at testing the knowledge of known algorithms studied at school. Obviously, the creative abilities of students are tested by other tasks of the EIA test. By structure, this cascade problem is not a linear problem, because subtasks 1 and 2 can be performed separately and independently of subtasks 3 – 6, which should be performed sequentially. Such problems will be called *cascade problems of mixed type*.

*Example # 3* (The Republic of Moldova 2019).On the picture $AEFG, AHIJ$ and $ABCD$ are squares. Using the data of the picture:




a) express the length of the side $AH$ by *x*;
b) find the area of the square $AHIJ$;
c) find the area of the shaded figure;
d)determine what percent of the area of *ABCD* is shaded.

This is a cascade problem of linear type. The student should use the answer to the previous subtask in order to solve the next task. Solving such problems educates students' attention, which is important when making decisions, including in life situations.

*Examples # 4* (Experimental exam, Ukraine 1989).

Given the right triangular prism $ABCA_1B_1C_1$. On the side edge $BB_1$ the point $M$ is chosen so that $BM = B_1M$.

a) Construct a section of the prism with a plane passing through the side of the base $AC$ and point $M$.

b) Prove that the resulting cross section is an isosceles triangle.

c) Calculate the perimeter of the section and the total surface area of the prism, if $BB_1$ equals to 8 cm, and the side of the base of the pyramid is 3 cm.

d) What is the ratio of the volumes of the parts of the prism into which it is divided by the cutting plane?

This is a cascading problem of mixed type, because it is possible to solve successive problems a) b) and d), without solving problem c), or problems a) and d), without solving problems b) and c).

*Example # 5* (Germany, Bayern 1995).

Points $A(-1;-1)$ and $C(4;4)$ are on the parabola $p_1$ defined by the equation $y = -x^2 + 4x + 4$. Parabola $p_2$ is defined by the equation $y = 0{,}5x^2 + bx + c$ $(b \in \boldsymbol{R}, c \in \boldsymbol{R})$ and also passes through the points $A$ and $C$.

1. Calculate the coordinates of the vertex $S_1$ of the parabola $p_1$ and plot this parabola in a rectangular coordinate system. For the drawing, take a unit segment of 1 cm and the axis within: $-3 \leq x \leq 6;\ -6 \leq y \leq 9$.
2. Find the unknown parameters $b$ and $c$ in the equation of the parabola $p_2$. Draw this parabola using the table of values for $x \in [-2;5]$ with step $\Delta x = 1$ in the coordinate system from the item 1. (*Expected intermediate answer*: $p_2: y = 0{,}5x^2 - 0{,}5x - 2$.)
3. The points $D_n(x;-x^2 + 4x + 4)$, belonging to the parabola $p_1$ and lying between the points $A$ and $C$, are the vertices of the triangles $ACD_n$. Draw the triangle $ACD_1$ for $x = 1$ in the coordinate system of item 1.
4. The straight line $q$ defined by the equation $y = 0{,}5x - 2{,}5$. Show that the line $q$ is tangent to the parabola $p_2$ and find the coordinates of the point of tangency $B$. (*Expected intermediate answer*: $B(1;-2)$).
5. Point $B$ is the vertex of the quadrilaterals $ABCD_n$, which have area $S(x)$. Of these quadrilaterals, the quadrilateral $ABCD_0$ has the largest area $S_{max}$. Find $S(x)$ and $S_{max}$. (*Expected intermediate answer*: $S(x) = -2{,}5x^2 + 7{,}5x + 17{,}5$.)
6. The quadrilatera l $ABCD_2$ is a trapezoid for which $BC||AD_2$. Draw this trapezoid in the coordinate system from the item 1 and find the coordinates of point $D_2$.

It is also the cascade task of mixed type. In some tasks, an intermediate answer is intentionally suggested so that a student who is unable to solve such a task can move forward using this intermediate answer.

**Methodological comments.**Analysis of the content of the above cascade problems revealed two main purposes for which they are proposed during assessing student achievement:

– *goal A* isto cover the widest possible area of mathematical concepts and their properties and thus more carefully check the amount of material learned by students during studying in secondary school (or to find out whether the students have mastered the entire program material or only part of it, find the gaps in their knowledge);

– *goal B* is using the proposed tasks to identify whether the student has formed the mathematical competencies provided by the program (graphic, analytical, computational, research, practical, etc.) and establish the level of their formation (intuitive, mandatory, sufficient, advanced, creative).

If, proposing a cascade problem, to pursue both goals at the same time, then such a task is "broad" in the scope of mathematical concepts and "deep" in the level of competencies, so it becomes for the student a task of increased complexity. This problem can only be done by students with a high level of mathematical training, and it takes a lot of time to solve it successfully, which is so often lacking during exams.

Another combination of both of these goals is possible: goal A dominates, and goal B acts as a subordinate to it, and vice versa. For instance, in the Example # 1 we present a cascade problem with six consecutive problems, which aims to test the amount of knowledge in algebra and the principles of analysis and stereometry (cube and its properties, a regular tetrahedron and its properties). It is clear that their solution will reveal the student's mathematical competencies (perhaps not all), but this goal is achieved indirectly, depending on the content of the tasks. Here, goal A prevails over goal B.

Other combinations of both goals are provided in Examples # 4 and # 5. These are cascade problems in which goal B dominates – to determine the level of mathematical competence of the student (graphic, computational, analytical, research, etc.). Therefore, one or more mathematical objects are given in the condition of the problem, and the amount of mathematical knowledge of the student is determined not directly, but indirectly.

Schematically, the relationship between goal A and goal B is shown in Figure 1.

| $A \Leftrightarrow B$ | $A \Rightarrow B$ | $B \Rightarrow A$ |
|---|---|---|
| Both goals are equally relevant | Goal A dominates over goal B. | Goal B dominates over goal A. |

**Figure 1.** The ratio of goal A and goal B.

Given the proposed hierarchy of objectives for cascade problems, we can say the following about their use and purpose.

1. If we want to identify the content and level of academic achievement of students in mathematics on a particular subject, section or course, then it is advisable to use cascade problems (their number will depend on the amount of educational material) of type $A \Rightarrow B$. There will be several consecutive tasks (subproblems) and their solution indicates the acquisition of mathematical knowledge by students (*content validity of the cascade problem*).
2. If we want to find out what mathematical competencies of the student are formed and at what level, it is advisable to use cascading problems of type $B \Rightarrow A$. Here the main thing will be to formulate consecutive tasks so that the corresponding mathematical competence is clearly manifested (draw, construct, calculate, justify, etc.) (*targetvalidityofthecascadeproblem*).
3. If it is necessary to reveal the knowledge of students of factual material in mathematics, and also the level of their mathematical competencies, then it is advisable to use cascading problems of type $A \Leftrightarrow B$.
4. Selections of such problems should be in school textbooks, collections of problems, tests in mathematics for thematic and final evaluation, SFA, EIA, final exams, etc.
5. One cascade problem allows you to reduce the number of tasks offered during the test of student's achievement. The student is less distracted by the various plots of conditions, and more focused on fulfilling the requirements of the tasks.
6. Consecutive tasks (subproblems) of the cascade problem can be varied according to the level of program requirements, which makes such tasks a good tool for implementing a differentiated approach to student learning.
7. Cascade problems in the means of control cannot be many. Their reasonable sufficiency should be maintained, because excessive amounts will only worsen the results of the inspection.

The above view on cascade problems and their purpose shows that the problem of competent and pedagogically balanced use of cascade problems in the educational process is extremely important and requires careful discussion among interested professionals in the field of mathematics teaching methods, as well as among practicing mathematics teachers. We conducted such a discussion through a statistical experiment in the form of a survey.

**Description and results of the statistic experiment.** To clarify the experience of using cascade problems of different types in the educational process, we conducted a survey of mathematics teachers in Ukraine and the Republic of Moldova. The survey was conducted online by filling out Google forms. It was attended by about 300 mathematics teachers from Ukraine and the Republic of Moldova. Here are the proposed survey questions with summary results and comments for them. In brackets, after the options for answering the questions with alternatives, the percentage of the total number of mathematics teachers in both countries who chose this option is given.

1. *"Cascade problem is a problem that contains common condition and an ordered list of tasks (subproblems) that should be done to complete it." Have you been familiar with this definition and this type of problems before?*
   a) Yes, I know the definition and this type of problems (30%).
   b) I am well aware of this type of problems, but I see the definition first time (45%).
   c) I heard something about this type of problems, but I see the definition first time (19%).
   d) I do not know this definition, and have never heard of this type of problems (6%).

As we can see, almost all teachers are familiar with this type of problems to varying degrees, and three quarters of them are well acquainted with these tasks. Interestingly, significantly more mathematics teachers in the Republic of Moldova are familiar with the definition of the cascade problem (63% of respondents) than in Ukraine (12%). At the same time, in Ukraine more than a third (34%) of respondents only something heard about cascade problems (in the Republic of Moldova this indicator is only 8.5%). This indicates a better theoretical awareness of this type of problem in the Republic of Moldova and provides additional motivation to familiarize teachers with cascade problems for Ukrainian methodologists.

2. *Have you used cascade problems in your practice and how often have you done so?*
   a) Yes, I constantly use cascade problems (18%).
   b) I use these problems, but infrequently (64%).
   c) I used these problems a long time ago, but I don't use them now (8%).
   d) I never used such problems (10%).

It should be noted that in Ukraine 25% of surveyed mathematics teachers do not use cascade problems in the educational process, and only 14% of respondents use them the educational process constantly. In the Republic of Moldova, 8.5% of respondents do not use cascade problems. Obviously, in the vast majority, these are the same Moldovan teachers that are unfamiliar with this type of task. 24% of Moldovan mathematics teachers constantly use cascade problems in their work, and 68% of them use them from time to time. Thus, we can conclude that cascading problems are more popular among mathematics teachers in the Republic of Moldova than in Ukraine.

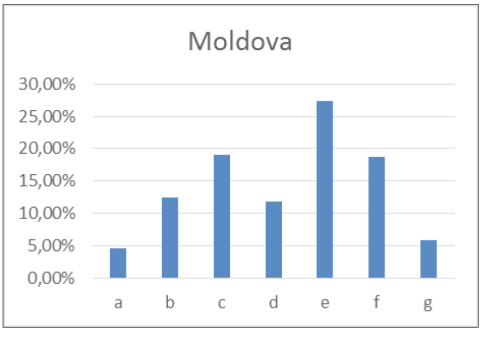
3. *Have you ever created cascade problems yourself and how often?*
   a) Yes, I constantly create such problems and use only my own cascade problems (3%).
   b) Yes, I often create such problems and use my own cascade problems rather than ready-made ones (13%).

c) Yes, I create such problems from time to time, but I use ready-made cascade problems more (48%).
d) I never created my own cascade problems, used ready-made ones only (36%).

Most teachers in Ukraine and the Republic of Moldova do not create cascade problems themselves, but use ready-made problems of this type. This is natural, because the creation of high-quality cascade problems requires careful methodological work and proper special training.

4. *At what stages of studying the topic or course of mathematics, in your opinion, it would be appropriate to use cascade problems? (itispossibletoch ooseseveralanswers)*
a) During explaining new material (8%).
b) During improving the ability to solve typical problems of the topic (21%).
c) During preparation for current testing and control works (19%).
d) Directly on current testing and control works (11%).
e) During the generalization and systematization of students' knowledge and skills (24%).
f) Directly on the final tests (11%).
g) On the final exams (6%).

The diagrams below show the differences between the spheres of using of cascade problems for Ukraine and the Republic of Moldova.


From the diagrams we can see that in Ukraine cascade problems are more used at the stages related to teaching mathematics (explaining new material, improving the ability to solve problems), while in the Republic of Moldova this type of problems predominates during the systematization of material and control actions (current tests and control works, final exams).

5. *Would you like to improve your skills in creating cascade problems?*
a) Yes, I would be happy to attend a course in the near future (59%).
b) Yes, but I do not consider it so important and urgent (31%).

c) No, there is no time and/or opportunity for that now (9%).

d) No, I think, I do not need it at all (1%).

It is obvious that the cascade type of problems in mathematics is interesting to teachers of both countries, because 90% of respondents would like to improve their ability to both create and apply problems of this type.

*6. What are the advantages of using cascade problems would you note?*

Respondents from Ukraine identified the following advantages of using cascade problems in the process of teaching mathematics:

– clarity and understandability of the formulation of the cascade problem for students, they know what and in what sequence to do;

– clear scheme for assessing cascade problem for mathematics teacher;

– opportunity for the student to take points even for incomplete solution of the cascade problem;

– it is clear the path of critical and logical thinking of students and the gradual formation of their mental actions;

– opportunity for the teacher to repeat important main knowledge and skills on the studied topic of the mathematics course by one cascade problem;

– assistance in preparation for the SFA and EIA in mathematics, because such problems are met there.

Mathematics teachers from the Republic of Moldova singled out some other advantages:

– cascade problems promote the development of memory, attention, logical and creative thinking of students;

– the ability to consider in one problem all the main typical problems of a particular topic of mathematics;

– the possibility to combine in one task problem from several topics of mathematics;

– using of several cognitive levels (understanding, application, generalization) in one problem

– cascade problems contribute to the implementation of intra-subject and inter-subject connections, as well as the implementation of the applied orientation of the school course of mathematics;

– such problems contribute to the formation and development of students' competencies;

– it is possible to achieve several educational goals with the help of one problem of cascade type;

– such problems increase the interest and motivation to study mathematics, develop memory and attention of students.

As we can see, the teachers of mathematics in both countries include as advantages of the cascading problems their complex nature, which contributes to the development of logical thinking and creativity of students, the formation of subject (mathematical) competence and many interdisciplinary competencies. In addition, such problems are convenient for teachers with the opportunity to cover in one problem the typical problems of one or more topics of the school course of mathematics.

*7. What disadvantages of using cascade problems would you note?*

Ukrainian mathematics teachers highlighted the following shortcomings of cascade problems:

– inability to continue solving after an error in the first stage;
– students have no experience in solving cascade problems and, consequently, fear of them;
– the difficulty of understanding the cumbersome condition of the cascade problem by the students;
– significant duration of solving cascade problems;
– inhibiting the development of initiative and creative thinking due to the algorithmic condition of the cascade problem;
– insufficient number of recommendations for teachers on the methodology of using cascade problems in educational process.

Respondents from the Republic of Moldova focused on the following shortcomings:

– inability to get the correct answer in the next stages of solving after an error in one of the previous ones;
– accomplishing cascade problems requires a lot of time from students;
– problems of this type are cumbersome and difficult to assess;
– not all students are positive about cascade problems;
– the difficulty of creating cascade problems by teachers;
– lack of methodological literature on the creation and use of such problems.

Thus, mathematics teachers in both countries consider the main disadvantages of cascade problems suppose their cumbersomeness, time consuming, and inability to obtain a correct final answer after error in solving previous subproblems. Another disadvantage is the lack of methodological and didactic literature on this topic.

**Conclusions**

Accomplished study allows us to formulate the following common conclusions:

– problems of cascade type are popular in the process of teaching mathematics at school, and therefore deserve increased attention from experts in the theory and methods of teaching mathematics;
– most teachers in Ukraine and the Republic of Moldova constantly use cascade problems at different stages of teaching mathematics – from explaining new material to monitoring student's achievements;
– using of cascade problems, depending on the purpose, allows us to identify the amount of material learned by students, and the level of their mathematical

competencies (computational, analytical, graphic, creative, etc.);
- the content of each individual subproblem of the cascade task should be clearly consistent with the dominant learning goal set by the teacher;
- in control works, in tasks with EIA and SFA on mathematics cascade problems should be just few (approximately 1 – 2), depending on the allocated time;
- the teaching community should be acquainted with the methods of creating and applying cascading tasks in the learning process;
- it is expedient to develop selections of cascade problems both for separate subjects of a school course of mathematics, and for carrying out repetition of educational material and final attestation of schoolchildren
- cascade problems in mathematics should become a research problem of the theory and methods of teaching mathematics.

In general, cascade problems contribute to the formation of students' mathematical culture and logical thinking, develop creativity and the ability to apply mathematical knowledge in real life. Solving such problems also contributes to the formation and development of interest and motivation of students to study mathematics, which is an important factor in the stage of modern education.Quite often life gives us tasks of the cascade type. The success of the project at all depends on the results of solving each stage of such a task. Therefore, students should be taught how to solve cascade problems at school, and at the final exams it is important to assess the level of formation of relevant competencies.

**ACKNOWLEDGMENTS**

The article is partially supported by the project RD-08-149/02.03.2022 of the Bishop K. Preslavski University Research Fund for 2022.

**REFERENCES**

AU, W., 2011. Teaching under the new Taylorism: High-stakes testing and the standardization of the 21st century curriculum. *Journal of Curriculum Studies*, **43**(1), 25 – 45.

BERLINER, D., 2011. Rational responses to high stakes testing: The case of curriculum narrowing and the harm that follows. *Cambridge Journal of Education*, **41**(3), 287 – 302.

Cabinet of Ministers of Ukraine, 2019. Elektronni petytsiyi: Vidmina ZNO v shkolakh, koledzhakh, tekhnikumakhta PTU [Electronic petitions: Abolition of EIA in schools, colleges, technical schools and vocational schools]. Retrieved from https://petition.kmu.gov.ua/kmu/Petition/View/2531 [in Ukrainian].

DVORETSKA, L.P., 2015. Zovnishnie nezalezhne otsiniuvannia z matematyky v Ukraini cherez pryzmu svitovoho dosvidu [External

Independent Assessment in mathematics in Ukraine through the prism of world experience]. *Naukovi zapysky CSPU*, **8**(III), 15 – 25. Retrieved from https://core.ac.uk/reader/228637440 [in Ukrainian].

KARP, A.P. (ed.), 2020. *Eastern European Mathematics Education in the Decades of Change*. Monograph. Switzerland. Springer.

LIASHENKO, O.I., RAKOV, S.A., 2008. Testovi tekhnolohii i monitoring v systemi osvity Ukrainy: stan i perspektyvy rozvytku [Test technologies and monitoring in the education system of Ukraine: state and prospects of development]. *Visnyk TIMO*, **11 – 12**, 67 – 70 [in Ukrainian].

LINGARD, B., W. MARTINO & G. REZAI-RASHTI, 2013. Testing regimes, account abilities and education policy: Commensurate global and national developments. *Journal of Education Policy*, **28**(5), 539 – 556.

MCLAUGHLIN, S., WEBBER, S.L., 2013. Final Project Evaluation USETI Legacy Alliance Project in Ukraine. Final Report. *International Business & Technical Consultants Inc.*

Ministry of Education of France, 2000. Kodeks obrazovaniya Francii [French Education Code] Retrieved from https://lexed.ru/zakonodatelstvo-ob-obrazovanii/kodifikatsiya-zakonodatelstva-ob-obrazovanii/kodeks-obrazovaniya-frantsii/ [in Russian].

Ministry of Education of Moldova Republic, 2021. Evaluări naţionale. Matematica. Clasa 12. [National evaluations. Mathematics. Class 12.] http://ance.gov.md/clasa-sesiunea-examen/clasa-12 [in Romanian].

MES of Ukraine, 2019. *Deyaki pytannia provedennia v 2021 rotsi zovnishnioho nezalezhnoho otsiniuvannia rezul'tativ navchannia, zdobutykh na osnovi povnoi zahal'noi serednioi osvity* [Some issues of conducting in 2021 an EIA of learning outcomes obtained on the basis of complete general secondary education]. Retrieved from http://search.ligazakon.ua/l_doc2.nsf/link1/RE33821.html?fbclid=IwAR1ZNLfb_K9-VHlw9010Tr9LT1IDvF-mxDhtiZaXYOoL4xMTD8BojZ6bp7 [in Ukrainian].

MES of Ukraine, 2021. Uchni 11-h klasiv u 2020/2021 navchalnomu roci zvilniayutsia vid prohodzhennya DPA [Graduates in 2020/2021 academic year will not pass SFA]. Retrieved from https://mon.gov.ua/ua/news/uchni-11-h-klasiv-u-20202021-navchalnomu-roci-zvilnyayutsya-vid-prohodzhennya-dpa-verhovna-rada-uhvalila-zakon [in Ukrainian].

Parliament of Moldova Republic, 2019. Codul educației al Republicii Moldova [Education Code of the Republic of Moldova]. Retrieved from http://www.legis.md/cautare/rezultate/110112 [in Romanian].

Parliament of Moldova Republic, 2011. Legea Educaţiei Naţionale [National Education Law]. Retrieved from https://www.edu.ro/sites/default/files/legea-educatiei_actualizata%20august%202018.pdf [in Romanian].

PROTASOVA, N.G., LUHOVYI, V.I., MOLCHANOVA, YU. O. et. al., 2012. *Osvitnia reforma v Ukraini: aspect derzhavnoho upravlinnia* [Education reform in Ukraine: public administration aspect]. Lviv: National Academy of Public Administration Publishing [in Ukrainian].

Rating Group Ukraine, 2018. *Dynamika stavlennia ukraiintsiv do ZNO* [Dynamics of Ukrainians' attitude to EIA]. Retrieved from http://ratinggroup.ua/research/ukraine/dinamika_otnosheniya_ukraincev_kvno_zno.html [in Ukrainian].

SAHLBERH, P., 2015. Finnish Lessons 2.0: What Can the World Learn from Educational Change in Finland? Second Edition. *Teachers College Press*.

SHKOLNYI, O. V., 2015. *Osnovy teorii ta metodyky otsiniuvannia navchalnykh dosiahnen z matematyky uchniv starshoi shkoly v Ukraini* [Basis of the theory and methodology of educational achievements assessing of senior school students]. Monograph. Kyiv. NDPU Publishing [in Ukrainian].

SHVETS, V.O., 1995. Vypusknyi ekzamen z matematyky [Final exam in mathematics]. *Postmetodyka*, #, **1**(8), 66 – 67.

SHVETS, V.O., BEVZ, V.G., SHKOLNYI, O.V., MATYASH, O, I., 2020. Ukraine: School Mathematics Education in the last thirty years. In A. Karp (Ed.), *Eastern European Mathematics Education in the Decades of Change*. Springer.

✉ **Prof. Dr. Oleksandr Shkolnyi**
ORCID ID: 0000-0002-3131-1915

✉ **Prof. Vasyl Shvets**
ORCID ID: 0000-0003-2084-1336
Department of Mathematics and Theory and Methodology of Math Teaching
National Pedagogocal Dragomanov University
9, Pyrogova St.
01601 Kyiv, Ukraine
E-mail: shkolnyi@ukr.net
E-mail: vasylshvets@ukr.net

✉ **Dr. Ion Akiri**
ORCID ID: 0000-0002-8874-2329
Department of Didactics of School Disciplines
Institute of Pedagogical Sciences
MD-2000 Chisinau, Moldova
E-mail: iakiri8@gmail.com