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# COMPUTER MODELS FOR PLAYING BILLIARDS USING FRONTAL SHOT

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**Abstract.** Computer models with pool table oriented to the school teaching of mathematics are presented. The pool table in them has a board with six pockets and twelve "diamonds", due to the expectation when working with these resources to form and apply not only mathematical knowledge, but also to master elements of the game of billiards. These computer models use a frontal impact, i.e. the cue ball hits the object ball in its center. The force of the shot, special shots, friction, etc. are not taken into account.

The computer models are developed with dynamic GeoGebra software. They are provided with free access in the Virtual Mathematics Laboratory, developed at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences.

*Keywords:* billiards; computer models; symmetry; eye-meter; STEM; digital competence; gaming; GeoGebra

#### Introduction

There are examples in Bulgarian education of the use of billiard tasks for teaching mathematics and information technology, especially in extracurricular activities (Jacobs 1987), (Dicheva, Nikolov & Sendova 1996). It is natural to create and use computer simulations on this topic in support of education. The billiard table in the provided computer models from billiard tasks from the competition "VIVA Mathematics with Computer" is a rectangle (Kenderov & Chehlarova 2020). A point represents a billiard ball, using the coordinates in the Cartesian coordinate system for its position.

In the computer models presented here, the pool table has a board. The six pockets (with black dots) and the twelve so-called "diamonds" are also marked. The reason for this is the desire to work with these resources to form and apply not only mathematical knowledge, but also to master elements of the game of billiards.

Computer models are described in which a frontal shot is used, i.e. the cue ball hits the object ball in its center. The force of the blow, special blows, friction, etc. are not taken into account.

Computer models have been developed with dynamic GeoGebra software (Hohenwarter, Hohenwarter & Lavicza 2009). They are provided with free

access in the Virtual Mathematics Laboratory (Chehlarova 2021), developed at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences<sup>1</sup>).

### 1. Computer models for direct hit

An object ball is placed on top of the cue stick on the pool table (Fig. 1). The ball is aimed. There is no cue ball on the table, because we are working on the frontal shot and are interested in the direction of the shot. The direction of a shot can be controlled using the red dot, which is the beginning of the vector (cue stick model). The aim is to direct the cue stick so that after the shot the ball falls into a pocket.

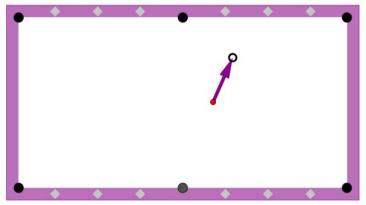


Figure 1. Computer model for direct hit<sup>2)</sup>

After pointing the rod, the correctness of the solution can be checked using the hide/show check mark (Fig. 2).

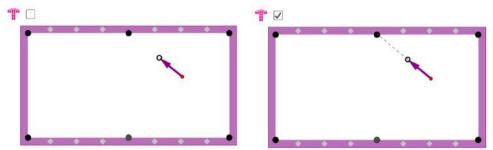


Figure 2. Actions in a computer model for direct hit

Three files are available at the specified address<sup>2)</sup>. In the first two files the ball is fixed, and in the third its place can be changed, i.e. to compile examples of this task.

The use of these files contributes to the development of the eye-meter. It is appropriate to set additional conditions for choosing a pocket in which to put the ball. This creates conditions both for learning the rules and for understanding the observed games and for developing strategies. Proximity to the pocket is important; the angle at which the ball is visible from it; the presence of barriers, i.e. of balls that limit "visibility". Conditions such as achieving appropriate positioning of the cue ball after the shot, as well as creating difficulties for the opponent when a foul is expected, etc. can also be discussed.

#### 2. Computer models for shot with one board

An object ball is placed on the pool table. The goal is to get it in a pocket after hitting single board. There are also several auxiliary points in the file, which serve to mark the place in the board where the ball will be hit after the shot.

It is appropriate to specify in advance in which pocket the target ball should enter. If the goal is for the given ball to enter the upper central pocket with one board, we mark with the auxiliary point the place in one of the boards where we will hit the ball (Fig. 3).

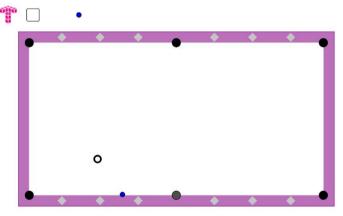


Figure 3. Computer model for shot with one board<sup>3)</sup>

The hide/show mark shows the cue stick, which is changed so that the dotted line showing the movement of the ball is directed to the already placed auxiliary point (Fig. 4 left). Of course, if the assessment is wrong, you can experiment and get a correct result (Fig. 4 right).

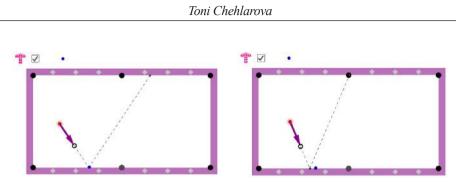


Figure 4. Checking the decision

We recommend that all possibilities for achieving the goal be indicated. For example, a blue dot to mark the place of shot on the board should be placed on both the right and left board (Fig. 5).

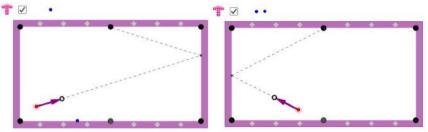


Figure 5. Completeness of decision

The topic of address<sup>4</sup> includes three files in which the position of the target ball can be changed: a simulator for monitoring its movement in the respective shot, which is guided by the model of the cue stick; a game in which help points are provided to mark the place of shot and a file in which the toolbar is shown. These tools can be used to draw with a pencil or sketch the movement with segments or with the tool "polyline" (Fig. 6), as well as for accurate construction using the button for axial symmetry.

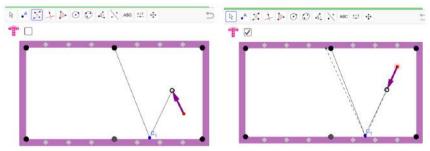
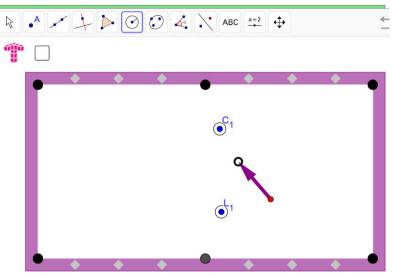


Figure 6. Additional constructions for finding a solution



Additional balls can be placed (by constructing points or circles), which will limit visibility and create limitations in solving the problem (Fig. 7).

Figure 7. Limit visibility with additional constructions

Models have also been developed in which the position of the ball is randomly generated (Fig. 8).

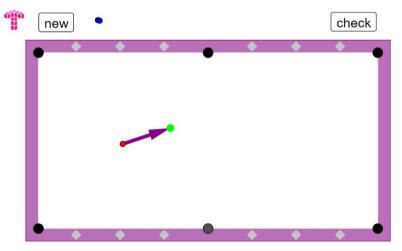


Figure 8. Randomly generated positions<sup>6)</sup>

### 3. Computer models for shot with two and three boards

The activities with computer models for two-board shot are similar to those for single-board shot. It is expected that, when working with these files, regularities will be noticed, for example, related to the parallelism of the lines (Fig. 9).

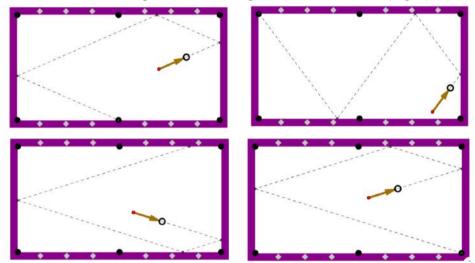


Figure 9. Three-board computer model

## 4. Placing a cue ball in a "9 balls" game

The task accompanying the file in Figure 10 is "Place the cue ball so that it can hit the center of the ball with the lowest number and score it directly or with hitting one board.

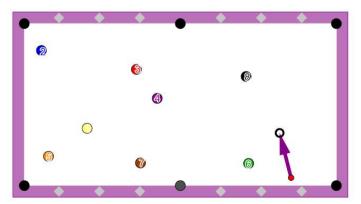


Figure 10. Beating ball task<sup>6)</sup>

This group of files also provides opportunities for actions under fixed initial conditions, as well as for creating new situations. A feedback for the trajectory of the ball is also provided (Fig. 11).

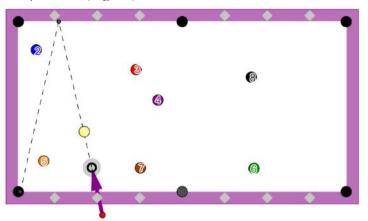


Figure 11. Feedback on trajectory

### Conclusion

The considered computer models on the one hand support the simultaneous development of mathematical and digital competence, and on the other - the inclusion of billiard games. The resources can also be used to create, through concretization or additional constructions, other didactic materials for teaching mathematics, physics, information technology and physical education. The work with the described files is related to types of angles (acute, right and obtuse), degree measure of angle, parallel lines, parallelogram, perpendicular lines, perpendicular from point to line, rectangle, supplementary angles, construction tasks, mutual position of point and line in the plane, mutual position of circles, axial symmetry, etc. The entertaining and gaming nature allows these resources to be used in arbitrary topics in mathematics in formal and non-formal education for propaedeutics or the application of knowledge, for the development of the eye-meter and for motivation for mathematical activity, which is a typical result of the application of both material and virtual educational games (Chehlarova 2021; Nikolova & Tuparova 2018; Pavlova 2019; Koneva & Shabanova 2021). The development of computer models with pool table is forthcoming, without the limitation for frontal shot.

### NOTES

- 1. http://cabinet.bg/
- 2. http://cabinet.bg/index.php?contenttype=viewarticle&id=359

- 3. http://cabinet.bg/index.php?contenttype=viewarticle&id=356
- 4. http://cabinet.bg/index.php?contenttype=viewarticle&id=357
- 5. http://cabinet.bg/index.php?contenttype=viewarticle&id=358
- 6. http://cabinet.bg/index.php?contenttype=viewarticle&id=363

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