Educational Technologies
Образователни технологии

SOME SIMPLE INTEREST MODELS

Tanka Milkova
University of Economics – Varna (Bulgaria)

Abstract. In this paper some aspects of financial mathematics and in particular some problems for simple interest are examined. As we know, the classical formula for simple interest is based on the assumption for constant initial investment and constant interest rate. The present study is mainly methodological and it examines three additional simple interest models – constant investment and variable interest rate, variable investment and constant interest rate, variable investment and variable interest rate. Some formulas are outlined – they can be used for educational purposes and for solving practical problems.

Keywords: simple interest; financial calculations; percentages

The problems concerning financial calculations compose one important part of mathematics – the so-called "financial mathematics". Some elements of financial mathematics are learned at school and in university education but with different level of depth. This seems quite logical, having in mind the countless applications of the financial operations not only in business but also in all the daily human activities. As an addition, we may mention the fact that in recent years one question is discussed very often – the question for the so-called financial literacy. The actuality of the financial math problems can be confirmed also by the already established tradition for organizing the International financial and actuary math Olympiad (Grozdev & al., 2018); (Shabanova & al., 2017; Nikolaev & al. 2017).

A basic concept in financial mathematics is the term interest (the money paid for the use of money lent or for delaying the repayment of a debt) (Dochev & al., 2010). It is well known that according to the basic principle for classifying the interest, it can be simple or compound. In the present paper, the research is focused on the methodology of simple interest and some specific problems are examined – with variable investment and variable interest. The study is mainly methodological because it concerns the deduction of some formulas which can be used in the educational process for financial mathematics. The models in the paper may be used for solving particular practical problems.

We will define in the beginning some well-known terms and notations, concerning simple interest (Dochev & al., 2010); (Capinski & Zastawniak, 2003), which will be used in the study.

Interest – an amount of money paid for the use of money lent or for delaying the repayment of a debt. The companies (the persons) pay interest for the lent money they use (as a credit) but they also receive money from the bank institutions for the money invested in them.

In the financial calculations the amount of money invested (or lent) is called basic or initial (basic capital or principal).

The total time, when the interest is paid, is called interest time.

The period of time in which beginning or end the interest is paid is called interest period. This period can be one year, six months, three months, one month etc. If nothing is said about the period, it is one year by default.

Usually, in practice there are different ways for determining the interest period and the length of the year in days. The number of days in the interest period can be defined accurately or approximately assuming that the length of a full month is 30 days. For the number of days in one year, we may take either the real length (365 or 366 days) or the approximate value 360 days (i.e. 12 months, 30 days each). In the present paper, we assume that the year consists of 360 days.

The interest which corresponds to 100 monetary units (e.g. 100 leva) is called interest rate (interest tax) – p.

A relative interest rate is the one which has the same proportion towards the annual rate as the corresponding time towards one year. If the interest is simple then the relative interest rate should be used.

In terms of convenience, we will use the interest for 1 monetary unit (1 lev) for one year – $i = \frac{p}{100}$.

In financial-credit practice, different types of interests are used.According to the method of calculation, the interest can be either simple (not accumulated) or compound (accumulated). The simple interest is calculated only for the basic capital. This interest is not added to the previous amount and does not bring additional interest for the following periods. The simple interest is used only for short-term financial operations, infinite deposits, payment accounts, etc.

In these financial calculations some basic elements are used:

1. Initial investment (basic capital), that is subject to interest calculation. This is the amount of money which is lent or borrowed and it is usually denoted by K.
2. The increased amount (or capital) for n interest periods. It is denoted by K_n and it includes the basic capital K plus the interest L ($K_n = K + L$).
3. Interest rate – p.
4. Interest period. It may be in years (n), months (m), days (d). If the interest period is in months then $n = \frac{m}{12}$. If the interest period is in days then $n = \frac{d}{360}$.

5. Interest – L. If the interest period is in years then for one year the interest is determined by $L = K.\frac{p}{100}$ or $L = K.i$ $\left(i = \frac{p}{100}\right)$. For n years the interest will be $L = K.n.\frac{p}{100}$ or $L = K.n.i$.

Based on the calculated simple interest we can determine the increased future value after n years:

$$(1)\quad K_n = K + L = K + K.n.\frac{p}{100} = K\left(1 + n.\frac{p}{100}\right) = K(1 + n.i),\ \left(i = \frac{p}{100}\right).$$

In this short fundamental simple interest model, it is assumed that the basic capital K and the interest rate p are constant values. But normally in practical situations, the simple interest calculations are held with variable initial capital and/or variable interest rate during the interest time interval.

Model 1. The basic capital K is constant, while the annual interest rate p varies in the corresponding time interval. The interest time interval consists of n days and it is split into different periods – n_1 days with interest rate p_1, n_2 days with interest rate p_2, n_3 days with interest rate p_3, and so on, till n_k days with interest rate p_k $(n_1 + n_2 + n_3 + ... + n_k = n)$.

A sketch of this example can be found in figure 1.

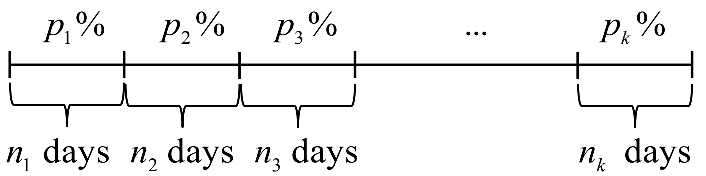

Figure 1

With all these conditions, the increased amount will be equal to:

$$K_n = K + K.\frac{p_1}{360}.\frac{n_1}{100} + K.\frac{p_2}{360}.\frac{n_2}{100} + K.\frac{p_3}{360}.\frac{n_3}{100} + ... + K.\frac{p_k}{360}.\frac{n_k}{100} =$$

$$= K.\left[1 + \frac{1}{36000}(p_1n_1 + p_2n_2 + p_3n_3 + ... + p_kn_k)\right] = K\left(1 + \frac{1}{36000}\sum_{i=1}^{k} p_in_i\right).$$

Here we assume that the annual interest rate is given but with identical steps we can deduce a formula if the given interest rate is not annual (for one day, one month, three months, six months and so on).

Model 2. The basic capital is variable and the interest rate p is constant for the whole period. We assume that for n_1 the basic capital is K_0, for n_2 days the previous amount is corrected by K_1, for n_3 days the previous amount is corrected by K_2, and so on, and finally for n_k days the previous amount is corrected by K_{k-1}. Again $n_1+n_2+n_3+\ldots+n_k=n$.

A sketch of this example can be found in figure 2.

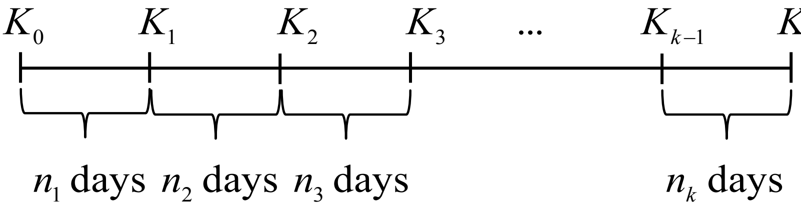

Figure 2

We assume that $K_0>0$, while K_i is the amount which is withdrawn or added at some stage. If $K_i<0$ the amount is withdrawn and if $K_i>0$, the amount is added, $i=1\div k-1$. Always $\sum_{i=0}^{j} K_i > \left|K_{j+1}\right|$, for each $j=1\div k-2$ and for each $K_{j+1}<0$. Then:

$$K_n=\sum_{i=0}^{k-1}K_i+K_0.\frac{p}{360}.\frac{n_1}{100}+(K_0+K_1)\frac{p}{360}.\frac{n_2}{100}+(K_0+K_1+K_2)\frac{p}{360}.\frac{n_3}{100}+$$

$$+\ldots+(K_0+K_1+K_2+\ldots+K_{k-1})\frac{p}{360}.\frac{n_k}{100}=$$

$$=\sum_{i=0}^{k-1}K_i+\frac{p}{36000}(K_0n_1+K_0n_2+K_1n_2+K_0n_3+K_1n_3+K_2n_3+\ldots+$$

$$+K_0n_k+K_1n_k+K_2n_k+\ldots+K_{k-1}n_k=$$

$$=\sum_{i=0}^{k-1}K_i+\frac{p}{36000}.\left[K_0.\sum_{i=1}^{k}n_i+K_1.\sum_{i=2}^{k}n_i+K_2.\sum_{i=3}^{k}n_i+\ldots+K_{k-1}.\sum_{i=k}^{k}n_i\right]=$$

$$=\sum_{i=0}^{k-1}K_i+\frac{p}{36000}.\sum_{j=0}^{k-1}K_j.\sum_{i=j+1}^{k}n_i.$$

Model 3. The basic capital is variable and the interest rate p is also variable during the period.

For deducing a general formula for calculating the increased amount according to this model we will use the following approach. The whole period will be divided into sub-periods, each of them with a constant interest rate, i.e. we assume that for m_1 days the interest rate is p_1, then for m_2 days the interest rate is p_2, for m_3 days the interest rate is p_3, and so on, until finally for m_T days the interest rate to be p_T. Each of these periods has a constant interest rate and variable capital, i.e. we can apply the strategy from Model 2. The situation is sketched in figure 3, figure 4, figure 5 and figure 6.

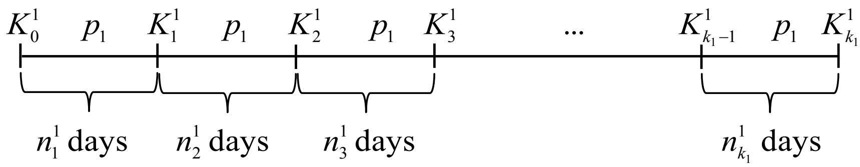

Figure 3

During the first period, the interest rate is p_1, K_0^1 is the initial amount of money, subject to the interest calculation, $K_1^1, K_2^1, ..., K_{k_1-1}^1$ are the consecutive changes in the capital (increasing or decreasing), as a result of the procedure, while the number of days in the interest time periods are correspondingly $n_1^1, n_2^1, n_3^1, ..., n_{k_1}^1$, where $\sum_{i=1}^{k_1} n_i^1 = m_1$.

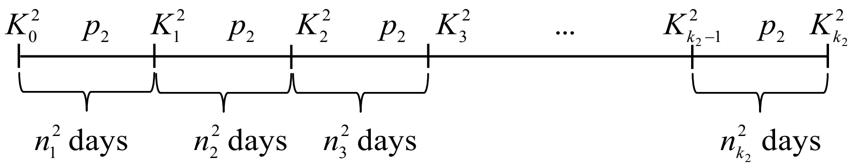

Figure 4

After m_1 days the interest rate will be changed from p_1 to p_2 and the second period starts. In this period K_0^2 is the initial amount of money, subject to the interest calculation, $K_1^2, K_2^2, ..., K_{k_2-1}^2$ are the consecutive changes in the capital (increasing or decreasing), as a result of the procedure, while the number of days in the interest time periods are correspondingly $n_1^2, n_2^2, n_3^2, ..., n_{k_2}^2$, where $\sum_{i=1}^{k_2} n_i^2 = m_2$, $K_0^2 = \sum_{i=0}^{k_1} K_i^1$.

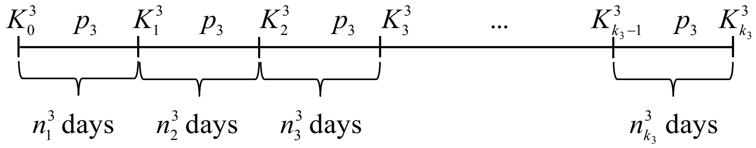

Figure 5

After m_2 days the interest rate will be changed from p_2 to p_3 and the third period starts. In that period K_0^3 is the initial amount of money, subject to the interest calculation, $K_1^3, K_2^3, ..., K_{k_3-1}^3$ are the consecutive changes in the capital (increasing or decreasing), as a result of the procedure, while the number of days in the interest time periods are correspondingly $n_1^3, n_2^3, n_3^3, ..., n_{k_3}^3$, where $\sum_{i=1}^{k_3} n_i^3 = m_3$, $K_0^3 = \sum_{i=0}^{k_2} K_i^2$.

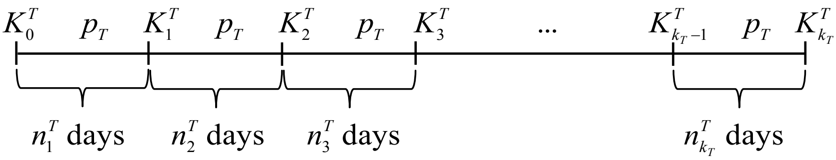

Figure 6

This procedure is repeated until the last moment when the interest rate is changed from p_{T-1} to p_T and the last period starts. Its length is m_T days. In that period K_0^T is the initial amount of money, subject to the interest calculation, $K_1^T, K_2^T, ..., K_{k_T-1}^T$ are the consecutive changes in the capital (increasing or decreasing), as a result of the procedure, while the number of days in the interest time periods are correspondingly $n_1^T, n_2^T, n_3^T, ..., n_{k_T}^T$, where $\sum_{i=1}^{k_T} n_i^T = m_T$, $K_0^T = \sum_{i=0}^{k_{T-1}} K_i^{T-1}$.

Also $\sum_{t=1}^{T} m_t = n$ days (or this is the full length of the interest time period) and

$K_{k_t}^t \equiv K_0^{t+1}$ and $K_0^{t+1} = \sum_{i=0}^{k_T} K_i^t$ for every $t = 1 \div T - 1$.

We assume that in the last moment no money will be withdrawn or added and the final future value will be equal to the basic amount with all the interest for the whole period. This is the formula we have to deduce. The interest for the period of m_T $(\forall \ t = 1 \div T)$ days will be equal to:

$$L_t = K_0^t . \frac{p_t}{360} . \frac{n_1^t}{100} + \left(K_0^t + K_1^t\right) . \frac{p_t}{360} . \frac{n_2^t}{100} + \left(K_0^t + K_1^t + K_2^t\right) . \frac{p_t}{360} . \frac{n_3^t}{100} + \ldots +$$

$$+ \left(K_0^t + K_1^t + K_2^t + \ldots + K_{k_t - 1}^t\right) . \frac{p_t}{360} . \frac{n_{k_t}}{100} =$$

$$= \frac{p_t}{36000} \left[K_0^t \sum_{i=1}^{k_t} n_i^t + K_1^t \sum_{i=2}^{k_t} n_i^t + K_2^t \sum_{i=3}^{k_t} n_i^t + \ldots + K_{k_t - 1} \sum_{i=k_t}^{k_t} n_i^t \right] =$$

$$= \frac{p_t}{36000} . \sum_{j=0}^{k_t - 1} K_j^t . \sum_{i=j+1}^{k_t} n_i^t .$$

Then the total interest for the whole period of n days will be:

$$L_n = \sum_{t=1}^{T} L_t = \frac{1}{36000} . \sum_{t=1}^{T} p_t . \sum_{j=0}^{k_t - 1} K_j^t . \sum_{i=j+1}^{k_t} n_i^t = \frac{1}{36000} . \sum_{t=1}^{T} \sum_{j=0}^{k_t - 1} \sum_{i=j+1}^{k_t} p_t K_j^t n_i^t$$

For the whole period of n days the basic amount will be:

$$\sum_{i=0}^{k_T - 1} K_i^T = K_{k_T}^T .$$

Thus, the final value in the last moment will be:

$$K_n = K_{k_T}^T + L_n = \sum_{i=0}^{k_T - 1} K_i^T + \frac{1}{36000} . \sum_{t=1}^{T} \sum_{j=0}^{k_t - 1} \sum_{i=j+1}^{k_t} p_t K_j^t n_i^t .$$

Conclusions: In the specialized scientific and educational literature the problems concerning simple interest are less examined than those concerning compound interest. In our opinion, the present paper will be an option to enrich the literature concerning the simple interest topic. Apart from this, it can be very useful for solving practical problems and also in the educational process for students at different levels. These models may improve their logical way of thinking and may help them to feel more confident in applications of the simple interest methods in specific situations.

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✉ **Dr. Tanka Milkova, Assoc. Prof.**
University of Economics – Varna
77, Kniaz Boris I Blvd.
9002 Varna, Bulgaria
E-mail: tankamilkova@ue-varna.bg