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# A CONTENT ANALYSIS OF THE RESULTS FROM THE STATE MATRICULATION EXAMINATION IN MATHEMATICS 

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#### Abstract

Abstact. This paper presents an analysis of the results from the State Matriculation Examination in Mathematics achieved by the students from the Science and Mathematics High School "Acad. S. Korolyov" within the period of 2012-2014.

Keywords: mathematics education


## Introduction

The reasons that prompted us to do this research are related to the great volume of empirical material accumulated so far and the possibility to use it to improve the learning outcomes of teaching Mathematics in the MPG. In this account, the analysis of the results has been carried out as the tasks have been grouped beforehand according
to the training material necessary for their solution. This approach can also be used to analyze the results at state level. Generally speaking, it is accepted that the inclusion of tasks with multiple choice answers are not informative enough concerning the students' achievements in Mathematics and Natural Sciences. Despite this fact, we suggest an approach that allows using these data through a content analysis and classifying the mistakes so that teaching Mathematics and Natural Sciences can be more successful.

## The study

In this study, the focus comes on analyzing the results of the test module which includes 20 tasks with multiple choice answers (four possible answers). The study includes a total of 234 works of pupils, presented by years as follows.

Table 1. Frequency distribution by years

| Year |  | Frequency |
| :---: | :---: | :---: |
| Valid | 2012 | 79 |
|  | 2013 | 85 |
|  | 2014 | 70 |
|  | Total | 234 |

The Table 2 shows the basic numerical characteristics of the total number of points of the module discussed by years.

Table 2. Numerical characteristics by years
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| Year | Mean | Median | Minimum | Maximum | Std. Deviation | Skewness | Kurtosis |
| :--- | :---: | :---: | ---: | ---: | ---: | ---: | ---: |
| 2012 | 40,7722 | 42,0000 | 13,00 | 50,00 | 7,01637 | $-1,354$ | 2,484 |
| 2013 | 44,7882 | 45,0000 | 29,00 | 50,00 | 4,66776 | ,- 802 | , 430 |
| 2014 | 45,1429 | 47,0000 | 28,00 | 50,00 | 5,17319 | $-1,306$ | 1,445 |
| Total | 43,5385 | 45,0000 | 13,00 | 50,00 | 6,01648 | $-1,425$ | 3,108 |

Despite the differences in the numerical characteristics by years, the suggested approach to analyzing the data according to the type of tasks allows to identify students‘ gaps and to seek ways to overcome them.

Algebraic errors are emphasized because the very format of the module requires calculations and comparison of the suggested numerical answers in order to obtain the final result (algebraic calculations), while the knowledge of the geometric facts is a necessity in view of solving the tasks, the latter being made significantly easier by means of the formulas and facts allowed to be used.

Table 3. Comparison of numbers

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Comparison of numbers | Task №1 (95) <br> Task №20 | Task №1 (95) <br> Task №2 (95) | Task №1 (5:1) |

Tasks 1. and 2. of 2012, Task 1. of 2013 and Task 1. of 2014 are successfully solved by most of the students.

Table 4. Transformation of mathematical expression

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Transformation of math- <br> ematical expression |  | Task №2 (100\%) | Task №2 (4:1) |

Task 2. of 2013 suggests numerical answers and they could easily check their answers.

Task 2. of 2014 the listed answers are equations that confuse the students leading to a large number of mistakes due to perplexity of checking them in an easy way.

Table 5. Square and be-square equation

| TOPIC | 2012 | 2013 | 2014 |
| :---: | :---: | :---: | :---: |
| Square and be-square <br> equation | Task №7 (13:3) | Task №9 (11:1) |  |

As a result of applying a cosine theorem, Task 9. of 2013 comes down to solving a square equation but still there are a lot of mistakes.

Task 7. of 2012 brings to light the fact that many of the graduates do not understand the theory of the be-square equation well.

Table 6. Inequalities

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Inequalities | Task №11 (11:5) <br> Task №4 (4:1) | Task №6 (2:1) | Task №4 (100\%) |

Task 4. of 2014 apparently did not prove to be difficult to the students because it can be solved by substituting the proposed answers in a square equation.

They found difficulty in solving Task 6 . of 2013 which is also a square equation because of the confusion due to answers as "no solution" and "every real number is a solution" where substitution is not possible.

Task 4. of 2012 requires the application of the interval method, hence the fact that $20 \%$ of the answers are wrong is discouraging.

The combination of inequality equation set by graphical representation of linear and square function in Task 11. of 2012 proved extremely difficult to the students. This is due to poor knowledge of the material concerning the graph of a function. Again, the wrong answers are evenly distributed, which speaks of wrong prediction.

Table 7. Viet's formula

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Viet's formula | Task №6 (9:1) | Task №5 (100\%) | Task №7 (100\%) |

The application of the Viet's formula in Task 5. of 2013 and Task 7. of 2014 shows a good knowledge of this material. The attested 10 percent of wrong answers in Task 6. of 2012 implies that the students should practice more.

The results obtained for the tasks involving Definition sets, i.e. Tasks 3. and 11. of 2014, Task 3. of 2013, are good. More mistakes are attested in the answers to Task 3. of 2012 which is probably due to the fact that a relation is suggested in the possible answer, so that the students could not ignore the wrong answers. The mistakes are a lot, furthermore the distraction due to the wrong answers implies wrong prediction.

Table 8. Definitive set

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Systems of equa- <br> tions |  | Task №12 (6:1) | Task №6 (100\%) |

Table 9. Systems of equations

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Systems of equa- <br> tions |  | Task №12 (6:1) | Task №6 (100\%) |

Task 6. of 2014 is marked by 100 percent of correct answers. The situation changes considerably when there is a requirement that the solutions should be presented in the Cartesian coordinate system which once again confirms the fact mentioned in reference to inequality equations - gaps in their knowledge of the coordinate system and graph of a function.

Table 10. Logarithm

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Logarithm | Task №5 (10:1) | Task №4 (100\%) | Task №5 (100\%) |

Task 5. of 2014, Task 4. of 2013 and Task 5. of 2012 are a direct application of the logarithm definition and they did well.

Table 11. Trigonometry

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Trigonometry | Task №8 (7:1) | Task №7 (100\%) | Task №8 (100\%) <br> Task №14 (100\%) |

The suggested tasks use the median formula and the property of the bisector (Task 17. of 2014, Task 8. of 2013) and the graduates were able to deal with them. Difficulty arose over Task 9. of 2012 in which the bisector property is not directly used.

Table 12. Progression

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Progression | Task № 13 (12:1) | Task №11 (4:1) | Task №13 (100\%) |

Task 13. of 2014 is a simple application of a formula that is given in the materials that can be used.

Task 13. of 2012 assumes that the students are able to deal with linear equalization systems.

Task 11. of 2013 reveals poorer results, provided there is no direct application of the mentioned formula. The attested 20 percent of mistakes in handling this kind of material seems troublesome.

Table 13. Number series

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Number series | Task №12 (1:9) | Task №10 (100\%) | Task №12 (4:1) |

Most of the students solved Task10. of 2013, whereas they failed in solving Task 12. of 2014, provided that the calculations are a little more complicated here. Concerning Task 12 of 2012, the first suggested answer proposes the formula (a) $3 n+2$, not for all natural numbers that give the residue 2 when divided by 3 . A terrifying $90 \%$ of the students chose this answer instead of the correct answer (b) $3 n-1$.

Table 14. Proportions

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Proportions | Task №20 (5:1) | Task №14 (100\%) <br> Task №13 (100\%) | Task №19 (100\%) <br> Task №9 (13:1) |

The proportion tasks in 2013 require direct application of proportions and the results show that students have done quite well. The same holds true for Task 19 of 2014 which treats relations among people.

In Task 9 of 2014 which is a direct application of Thales' theorem a lot of mistakes can be found due to confusion of the relation.

Task 20 of 2012 is a geometric task in which there is a combination of Thales' theorem and a bisection proved to be difficult to the students when they had to model the proportion, as evidenced by the high percentage of errors.

Table 15. Right-angled triangle

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Right-angled <br> triangle | Task №19 (7:1) | Task №16 (100\%) |  |
|  | Task №17 (11:2) | Task №19 (6:1) | Task №10 (100\%) |
|  | Task №16 (7:2) | Task №18 (5:1) | task №20 (100\%) |

The suggested 8 tasks show that the students did not face difficulty in solving a rectangular triangle. However, they encountered difficulties when they had to find it as an element of a rectangle-Task 16, 17, 19, 2012; 17, 18, 2013.

Table 16. Other triangles

| TOPIC | TАSKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Other triangles | ЗАДАЧА №9 (4:1) | ЗАДАЧА №8 (100\%) | ЗАДАЧА №17 (100\%) |

Table 17. Sine and cosine theorem

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Sine and cosine <br> theorem | Task №18 (7:2) | Task №9 (11:1) <br> Task №15 (10:1) | Task №18 (11:3) |

The property of the bisector in an isosceles triangle is applied correctly when it is applied directly. There are a lot of mistakes in task 8 of 2013 as it is an element of the task. Task 20 of 2014 and task 17 of 2014 are direct application of the median, where the formula is given in an appendix.

Task 18 of 2012, Task 9 and 15 of 2013, and Task 18 of 2014 demonstrate a significant amount of wrong answers, indicating poor implementation of this theory.

Table 18. Combinatorics

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Combinatorics | Task №14 (100\%) | Task №20 (9:5) | Task №16 (1:1) |

Table 19. Statistics

| TOPIC | TASKS |  |  |
| :---: | :---: | :---: | :---: |
|  | 2012 | 2013 | 2014 |
| Statistics | Task №15 (6:1) |  | Task №15 (11:1) |

Task 14 of 2012 is a simple application of the Cartesian work of multitudes, hence this part proved quite easy to the students, whereas in reference to Task 20 of 2013, where two combinational formulas /sets of combinations/ are required, the results reveal gaps in building the correct model. This is further confirmed by Task 16 of 2014, wherein the application of both variations and combinations leads to serious confusion and more wrong answers, and the prevalent number of wrong answers again shows that the students did not seek logical reasoning for their answers.

Statistics is used in Task 15 of 2012 and Task 15 of 2014 which require direct application of the definitions of median, mode and mean. Despite the high percentage of correct answers, the mistakes made in a task involving elementary calculations speak of the lack of knowledge of definitions.

## Conclusion

In conclusion, it must be pointed out that even based on the test module that is not the most appropriate for assessing the mathematics results a sufficient number of conclusions can be drawn about the mathematical learning difficulties and gaps and recommendations for their correction and eradication. The research will be further conducted with the analysis of the other two modules for the period from 2012 to 2016.

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